Flux–Profile Relationships in the Stable Boundary Layer

A Critical Discussion

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Index

- ❑ Scientific context and problem shaping
- ❑ Published flux-profile relationship introduction
- ❑ Consistency of flux-profile relationships with the Monin-Obukhov similarity theory (MOST)
- ❑ How can we achieve a better formulation ?

Scientific context

Why are flux-profile relationships crucial ?

a. Parametrization of surface fluxes of momentum and heat

b. Validation of MOST extension to very stable conditions

c. Pollutant dispersion modelling

d. Numerical weather and climate prediction

Open problems

a. Generally accepted functions are still missing

b. Measuring stable and very stable boundary layer (SBL) when turbulence parameters are close to instrumental uncertainties

c. Disentangle turbulence from non-turbulent motions – internal gravity waves, Kelvin-Helmholtz shear instability, low-level jets, sub-meso motions

Are the flux-profile relationships proposed in literature consistent with MOST ?

Problem shaping

$$\frac{dU}{dz} = \frac{u_*}{kz}\phi_m(\zeta)$$

$$\frac{d\theta}{dz} = \frac{T_*}{kz}\phi_h(\zeta)$$

$$U = \frac{u_*}{z}\left[\ln\frac{z}{z_0} - \Psi_m(\zeta)\right]$$

$$\theta = \theta_0 + \frac{T_*}{z}\left[\ln\frac{z}{z_0} - \Psi_h(\zeta)\right]$$

with:

$$\Psi_{m,h}(\zeta) = \int_{\zeta_{0,0h}}^{\zeta} dx\,\frac{1-\phi_{m,h}(\zeta)}{x}$$

$$T_* = -\frac{\overline{w'\theta'}}{u_*} = -\frac{H_0}{u_*} \qquad \zeta = \frac{z}{L} \qquad L = -\frac{\theta_r}{kg}\frac{u_*^3}{H_0}$$

$$Ri = \frac{g}{\theta_r}\cdot\frac{\frac{\partial\theta}{\partial z}}{\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2} = \zeta\frac{\Phi_h(\zeta)}{\left(\Phi_m(\zeta)\right)^2} \xrightarrow[\zeta\to\infty]{} \infty$$

$$Rf = \frac{g}{\theta_r}\cdot\frac{\overline{w'\theta'}}{\overline{u'w'}\left(\frac{\partial U}{\partial z}\right) + \overline{v'w'}\left(\frac{\partial V}{\partial z}\right)} = \frac{\zeta}{\Phi_m(\zeta)} \xrightarrow[\zeta\to\infty]{} \sim 0.2$$

Richardson numbers and ζ are all stability parameters, and are linked together by flux-profile relationships

Considering a stationary SL the TKE balance can be used to demonstrate that Rf reaches an asymptotic value when $\zeta \to \infty$, while Ri does not.

Problem shaping

Similarly, kinematic heat flux and the scale temperature can be expressed as a function of flux-profile relationships

$$H = -H_0 = \frac{\theta_r}{kzg}\zeta\left[\frac{kz \cdot dU/dz}{\Phi_m(\zeta)}\right]^3$$

$$T_* = \frac{kz\theta_r}{g}\zeta\left(\frac{dU/dz}{\Phi_m(\zeta)}\right)^2$$

$$H = \left(\frac{g}{\theta_r}\right)^{1/2}(kz)^2\left(\frac{d\theta}{dz}\right)^{3/2}\zeta^{-1/2}\Phi_h^{-3/2}(\zeta)$$

$$H = -H_0 = \frac{\theta_r}{kzg}\zeta\left[\frac{kU}{\ln(z/z_0) - \Psi_m(\zeta)}\right]^3$$

$$T_* = \frac{\theta_r}{kzg}\zeta\left[\frac{kU}{\ln(z/z_0) - \Psi_m(\zeta)}\right]^2$$

$$H = \sqrt{\frac{kzg}{\theta_r}}\zeta^{-1/2}\left(\frac{k(\theta - \theta_0)}{\ln(z/z_{0h}) - \Psi_h(\zeta)}\right)^{3/2}$$

$$H_* = H\frac{g}{(kz)^2\theta_r(dU/dz)^3} = \frac{\zeta}{\Phi_m^3(\zeta)}$$

$$T_{**} = \frac{g}{kz\theta_r}(dU/dz)^{-2}T_* = \frac{\zeta}{\left(\Phi_m(\zeta)\right)^2}$$

$$H_{**} = H\left(\frac{g}{\theta_r}\right)^{-1/2}(kz)^{-2}\left(\frac{d\theta}{dz}\right)^{-3/2} = \zeta^{-1/2}\Phi_h^{-3/2}(\zeta)$$

$$H_+ = \frac{zg}{k^2}\frac{1}{\theta_r U^3}H' = \frac{\zeta}{[\ln(z/z_0) - \Psi_m(\zeta)]^3}$$

$$T_{++} = \frac{zg}{\theta_r U^2}T_* = \frac{\zeta}{[\ln(z/z_0) - \Psi_m(\zeta)]^2}$$

$$H_{++} = \sqrt{\frac{\theta_r}{kzg}}\left(\frac{1}{k(\theta - \theta_0)}\right)^{3/2}H = \frac{\zeta^{-1/2}}{(\ln(z/z_{0h}) - \Psi_h(\zeta))^{3/2}}$$

Flux-profile formulations

$$\phi_m(\zeta) = 1 + \beta_m \zeta$$

$$\phi_h(\zeta) = 1 + a\zeta + b\zeta^2$$

$$\beta_m = 5,\ a = 4,\ b = 1.25$$

Reference

Zilitinkevich e al., 2010
Kouznetsov e Zilitinkevich, 2010

Theoretical arguments and LES simulations – neutral condition

Asymptotic Rf and Ri behaviour as expected

$$\phi_m(\zeta) = 1 + \beta_m \zeta$$

$$\phi_h(\zeta) = \alpha_h^{-1}(1 + \beta_h \zeta)$$

$$\beta_m = 5.3,\ \beta_h = 8,\ \alpha_h^{-1} = 0.95$$

Businger-Dyer

Businger e al., 1971
Dyer et al, 1974, 2010

Weakly stable condition, 15-min averaged data

Critical values for both Rf and Ri

$$\phi_m(\zeta) = 1 + a\zeta + b\zeta[1 + c - d\zeta]e^{-d\zeta}$$

$$\phi_h(\zeta) = 1 + a\zeta\left[1 + \frac{2}{3}a\zeta\right]^{\frac{1}{2}} + b\zeta[1 + c - d\zeta]e^{-d\zeta}$$

$$a = 1,\ b = 0.667,\ \mathrm{c} = 5,\ \mathrm{d} = 0.35$$

Beljaars–Holtslag

Beljaars and Holtslag, 1991

France and Holland, 10-min averaged data

Asymptotic Rf and Ri behaviour as expected

Flux-profile formulations

$$\phi_m(\zeta) = 1 + a\left(\frac{\zeta + \zeta^b(1+\zeta^b)^{\frac{1-b}{b}}}{\zeta + (1+\zeta^b)^{1/b}}\right)$$

$$\phi_h(\zeta) = 1 + c\left(\frac{\zeta + \zeta^d(1+\zeta^d)^{\frac{1-d}{d}}}{\zeta + (1+\zeta^d)^{1/d}}\right)$$

$a = 6.1,\ b = 2.5,\ \mathrm{c} = 5.3,\ \mathrm{d} = 1.1$

No critical values

CASES-99

Chenge and Brutsaert, 2005

Mid-latitude (Kansan, USA), 60-min averaged data

Universal functions for wind and temperature profile (not shown) lead to similar Rf and Ri asymptotic behaviour

$$\phi_m(\zeta) = 1 + \frac{a_m\zeta}{(1+\beta_m\zeta\,)^{2/3}}$$

$$\phi_h(\zeta) = Pr_0\left(1 + \frac{a_h\zeta}{1+\beta_h\zeta}\right)$$

$Pr_0 = 0.98,\ a_m = a_h = 5,\ b_m = 0.3,\ b_h = 0.4$

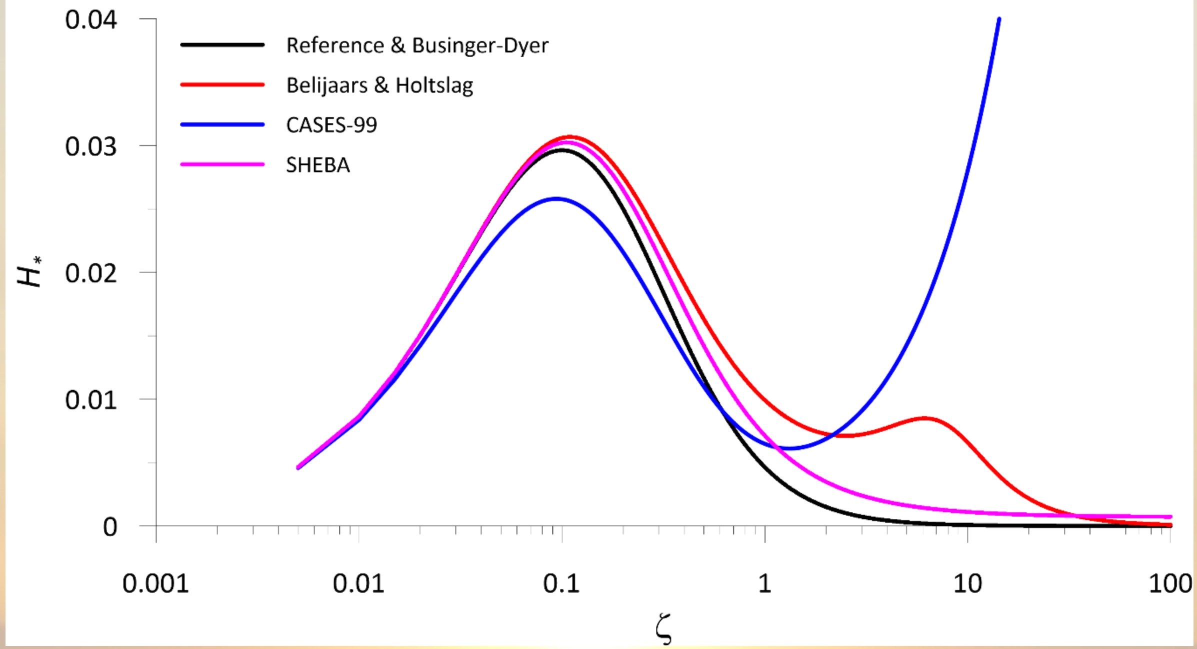
No critical values

SHEBA – new formulation

Gryanik et al., 2020

Arctic ocean, stable and very stable conditions ($0 < \zeta < 100$), 60-min averaged data spectrally corrected to isolate higher frequencies

Results

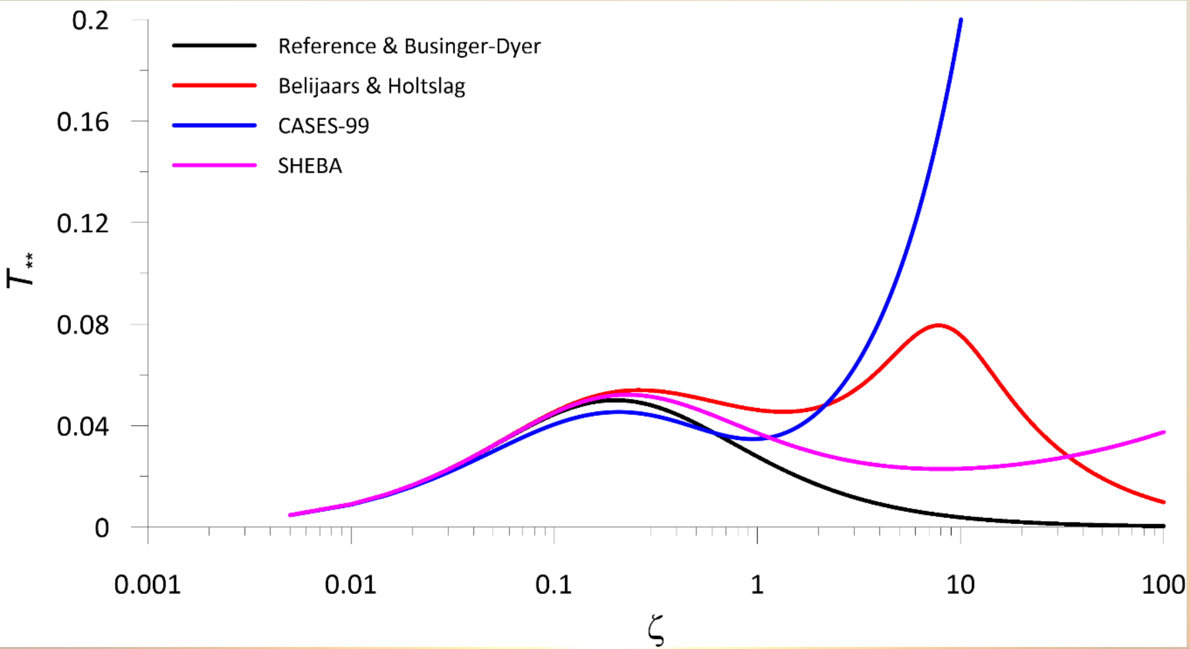


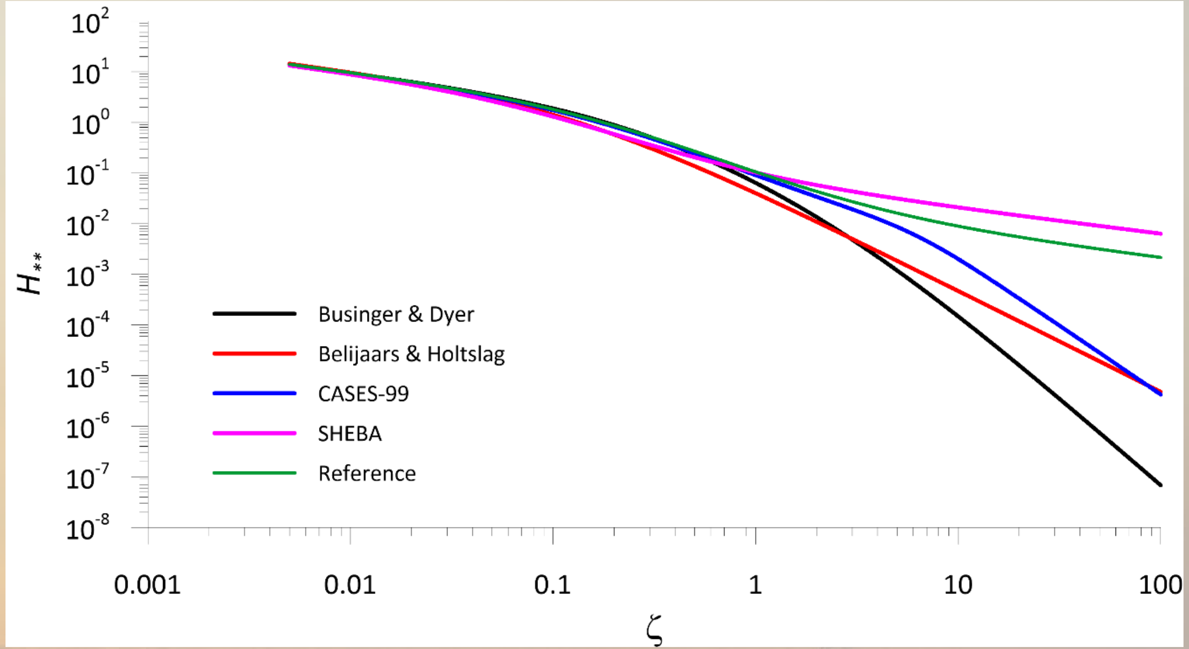
$$H_* = \frac{\zeta}{\Phi_m^3(\zeta)}$$

- SHEBA Φ_m leads to a trend similar to that expected, with a single maximum at $\zeta \cong 0.1$ and $H_* \to 0$ when $\zeta \to 0$ and $\zeta \to \infty$. We were not able to reproduce the local minimum at the large stability values ($\zeta \cong 80$) reported in literature.
- Beljaars–Holtslag Φ_m presented a first maximum at $\zeta \cong 0.1$, but beyond that it did not decrease monotonically as expected, reaching a second maximum at $\zeta \cong 6.1$.
- CASES-99 Φ_m performance was disappointing, leading to a function that increased as stability increased.

Results

Such a monotonic trend completely contradicted the expected behaviour !



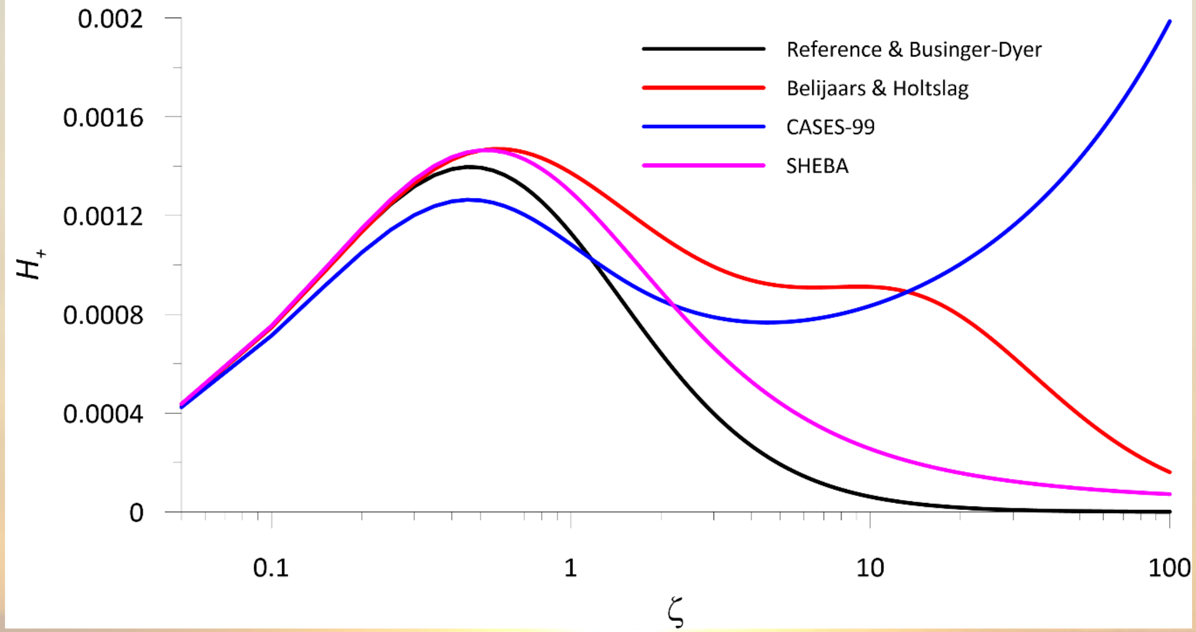


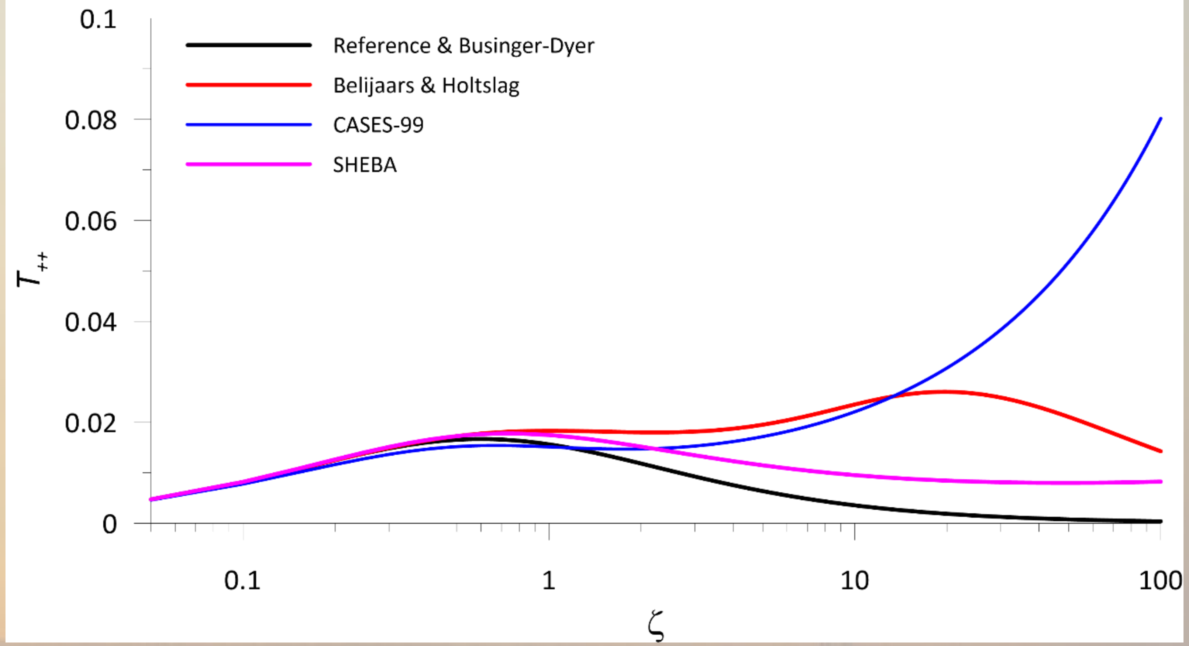
$$T_{**} = \frac{\zeta}{\left(\Phi_m(\zeta)\right)^2}$$

$$H_{**} = \zeta^{-1/2}\Phi_h^{-3/2}(\zeta)$$

Results

H_{++} trends (not shown) are analogous to those of H_{**}.





$$H_+ = \frac{\zeta}{[\ln(z/z_0) - \Psi_m(\zeta)]^3}$$

$$T_{++} = \frac{\zeta}{[\ln(z/z_0) - \Psi_m(\zeta)]^2}$$

Discussion

Summary

- Both the reference and the four experimentally determined formulations for Φ_m, Φ_h Ψ_m, and Ψ_h are expected to lead to similar outcomes, but our analysis shows otherwise
- All Φ_m and Ψ_m are capable of describing the expected H and T_* trends for subcritical values of ζ, but three out of four formulations lead to unreliable results when stability increases.
- When considering Φ_h and Ψ_h functions, all formulations fail

Possible explanations

- All campaigns did not take into account nonturbulent motions, that are expected to become more significant as stability increases
- None of the analysed formulations considered the the possible presence of self-correlation, which may affect the regression analysis.
- All the formulations considered here were obtained independently of each other, i.e. neglecting any possible physical constraints or relation between them – but MOST requires for universal functions to be congruent with all the similarity relationships in which they are included.

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Discussion

$$Ri = \zeta \frac{\Phi_h(\zeta)}{\left(\Phi_m(\zeta)\right)^2} = \Im_{R_i}(\zeta)$$

- Not affected by self-correlation
- Common constrain for both $\Phi_m(\zeta)$ and $\Phi_h(\zeta)$, along with the absence of a critical value

$$Rf = \frac{\zeta}{\Phi_m(\zeta)} = \Im_{Rf}$$

- It allows to determine $\Phi_m(\zeta)$
- It reaches a critical value (constrain) when $\zeta \rightarrow \infty$
- Self-correlation need to be addressed