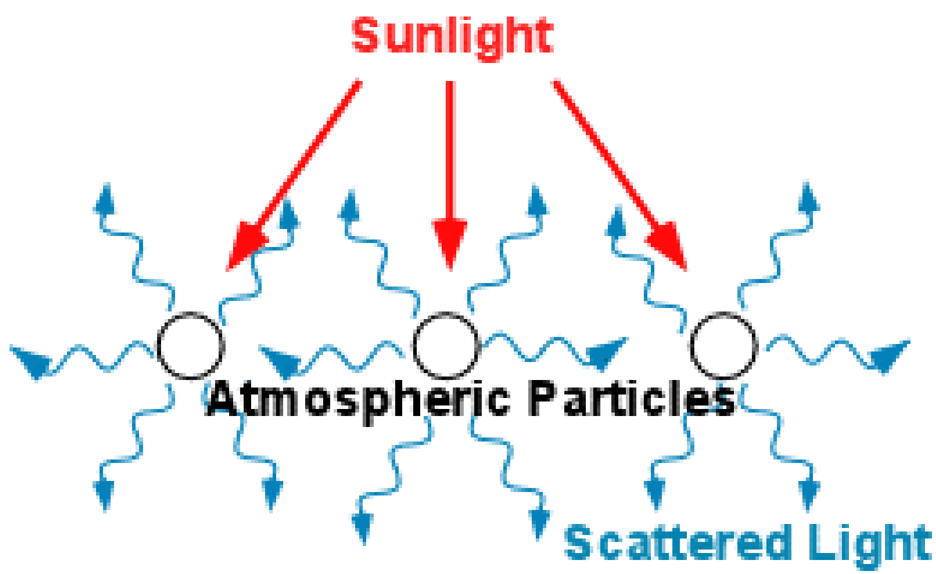
Lesson 9:
ELASTIC AND INELASTIC SCATTERING



Course: Laboratory of Atmospheric Remote Sensing
Laurea Magistrale in Atmospheric Science and Technology

Content

- Scattering: definition, characteristics
- Scattering of electromagnetic radiation
- Rayleigh scattering: Raman scattering, scattering from electric dipole, polarizability, polarization influence, intensity in the plane perpendicular and parallel to the incident electric field, intensity of non-polarized radiation
- Blue sky
- Phase function for Rayleigh scattering
- Scattered flux, scattering cross section, polarizability
- Mie scattering

<u>Reading material:</u>

K. N. Liou, An Introduction to Atmospheric Radiation, Academic Press, 2002 (Chapter 3)

Scattering (1)

Scattering is the process by which "small particles suspended in a medium of a different index of refraction diffuse a portion of the incident radiation in all directions".

With scattering, there is no energy transformation, but a change in the spatial distribution of the energy.

Scattering, along with absorption, causes attenuation problems with radar and other measuring devices.

$\text{Particle} < \frac{1}{10}\lambda \quad \Longrightarrow \quad$ RAYLEIGH SCATTERING

$\frac{1}{10}\lambda < \text{Particle} < \lambda \Longrightarrow \quad$ MIE SCATTERING

$\text{Particle} > \lambda \Longrightarrow \quad$ GEOMETRIC OPTICS

Scattering (2)

In addition to the absorption process, in which the photon increases the energy of the molecule, the atmosphere can interact with electromagnetic radiation through scattering or diffusion.
In this case, the photon undergoes a change in the direction of propagation and/or wavelengths without changing the energy of the molecule.
In the case in which the wavelength of the scattered radiation remains unaltered, the scattering is called elastic, otherwise inelastic (Raman, fluorescence, etc.).
Scattering deflects the solar radiation in all directions (daylight) and partly contributes to the albedo terrestrial.

The intensity of the scattering depends above all on the ratio between the size of the diffuser and the wavelength of the incident radiation, called **size parameter** and expressed as

$$x = \frac{2\pi a}{\lambda}$$

where a is the equivalent radius of the diffuser.

Scattering (3)

The molecules that form the gaseous part of the atmosphere produce scattering that is independent of the composition, because, being much smaller than the wavelengths affected by the radiative transfer, the changes in shape and size do not induce changes in the scattering process

The size of the aerosol component (including cloud droplets, raindrops, hail grains and snowflakes) vary in size over a large range of values that include all the wavelengths involved in the radiative transfer.

In this case also the composition and the shape influence the scattering characteristics.

The theoretical description of scattering is based on different models depending on the orders of magnitude of the size parameter.

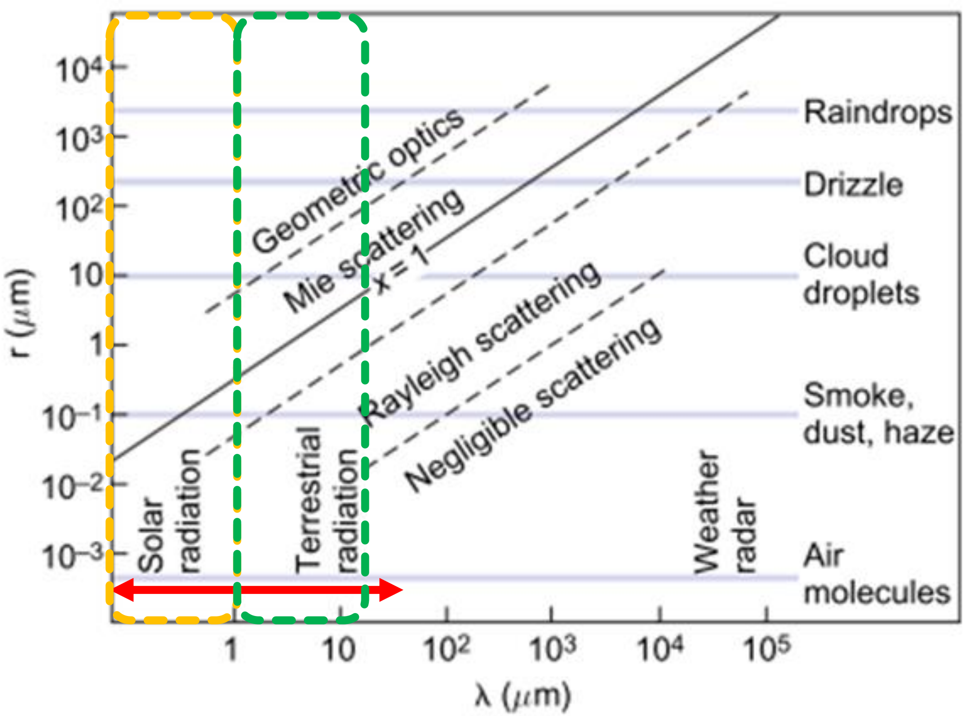
$x<1$ → Rayleigh electromagnetic dipole theory

$x\geq1$ → Mie scattering, theory that calculates scattering from spherical diffusers of uniform composition

$x\gg1$ → optical geometry

Scattering of electromagnetic radiation (1)

In this graph, the various types of diffusers are related to the most suitable scattering theory as a function of their <u>wavelength</u>.
For diffusers with dimensions much smaller than the wavelength ($x < 10^{-3}$) the scattering is negligible.
The horizontal axis also indicates the spectral ranges of solar and terrestrial radiation and the wavelengths used by meteorological radars for the study of clouds and precipitation.



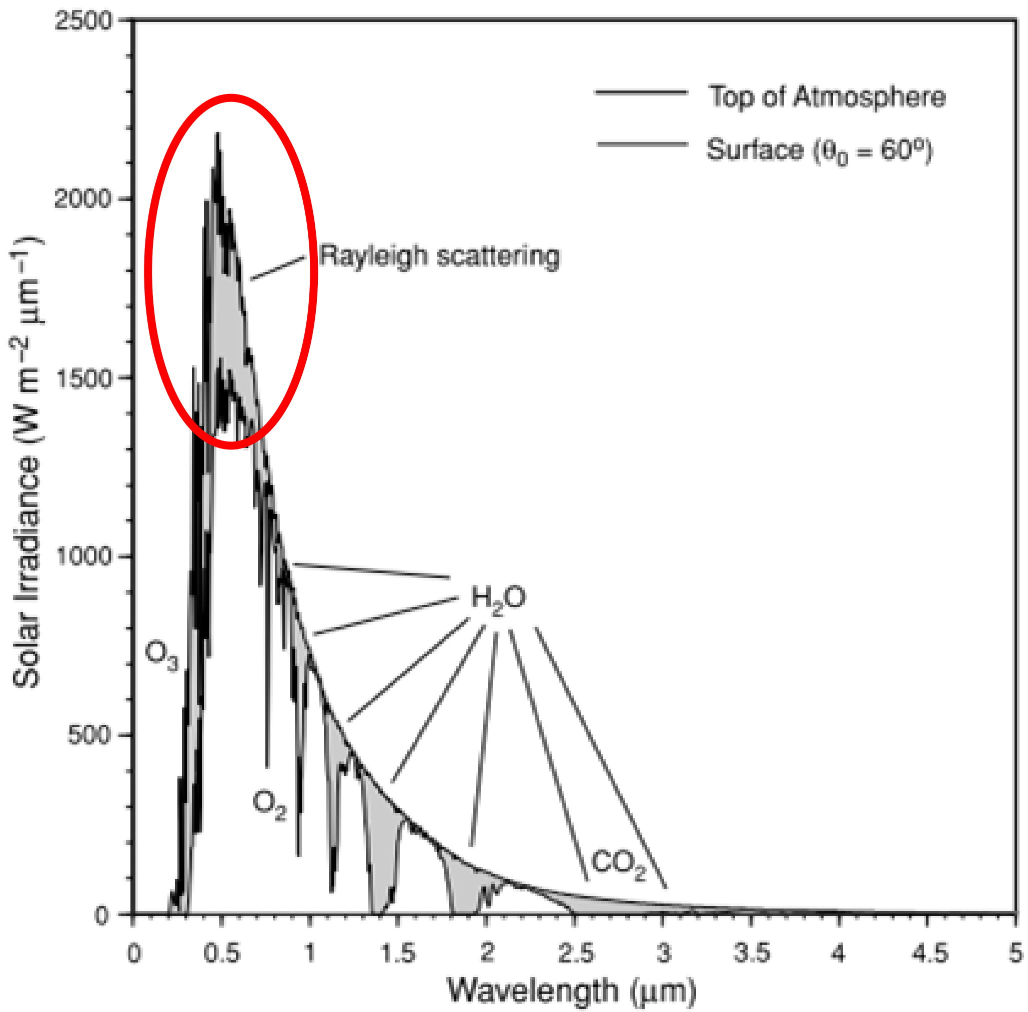
Solar radiation (λ< 1µm):

Air molecules → Rayleigh
Aerosols → Mie
Clouds and precipitation → geometric

Terrestrial radiation (λ≈10µm):

Air molecules → negligible
Aerosols →Rayleigh
Clouds → Mie
Precipitation → geometric

Scattering of electromagnetic radiation (2)

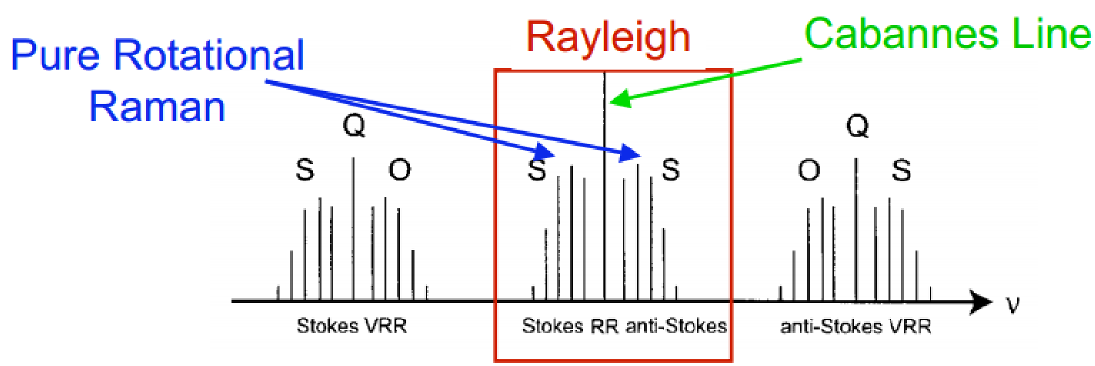


Rayleigh scattering

1. **Rayleigh scattering** is referred to the elastic scattering from atmospheric molecules (particle size is much smaller than 1/10 of the wavelength), i.e., scattering with no apparent change of wavelength, although still undergoing Doppler broadening and Doppler shift.

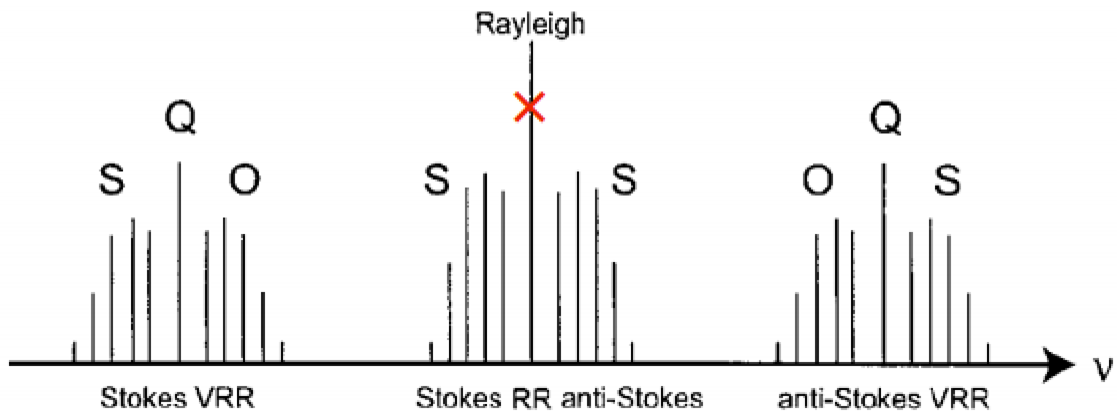
The theory of scattering from diffusers smaller than the wavelength was developed by Rayleigh (1871) based on the properties of the electric dipole.

However, depending on the resolution of detection, Rayleigh scattering can consist of the Cabannes scattering (really elastic scattering from molecules) and pure rotational Raman Scattering.



Raman scattering

Raman scattering is the inelastic scattering with rotational quantum state or vibration-rotational quantum state change as a result of scattering.

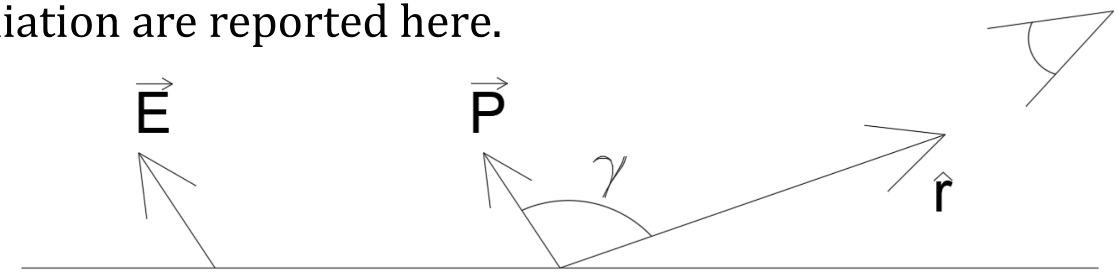


Separation between Cabannes line and the nearest pure rotational Raman (O_2 and N_2) is about 10 cm^{-1}=300 GHz.

For vibration-rotational Raman, Q branch is not a single line, but consists of many lines with very small separations, <0,1cm^{-1}=3 GHz, due to the different splitting of the rotational levels in the lower and upper vibrational levels.

Scattering from electric dipole (1)

A schematic treatment of the theory can be found in Liou book (chapter 3.3.1.1). Only the results in the case of polarized incident radiation are reported here.



$\overrightarrow{E_0}$= homogeneous electric field due to the incident radiation
$\overrightarrow{P_0}$ = dipole momentum
γ = angle between dipole direction and scattering direction
α = polarizability
r = distance
c = light speed

$$\overrightarrow{E_i} = \overrightarrow{E_0} e^{-ik(r-ct)}$$

$$\vec{P} = \alpha \overrightarrow{E_i} = \alpha \overrightarrow{E_0} e^{-ik(r-ct)}$$

$$\boxed{\overrightarrow{P_0} = \alpha \overrightarrow{E_0}}$$ POLARIZABILITY α OF A SMALL PARTICLE

Scattering from electric dipole (2)

$$\left|\vec{E}\right| = \frac{1}{c^2}\frac{1}{r}\frac{\partial^2|\vec{P}|}{\partial t^2} sin\gamma$$

Electrical field generated by induced dipole (far-field)

Introducing k = wave number (2π/λ):

$$\frac{\partial\left|\vec{P}\right|}{\partial t} = kc|p|$$

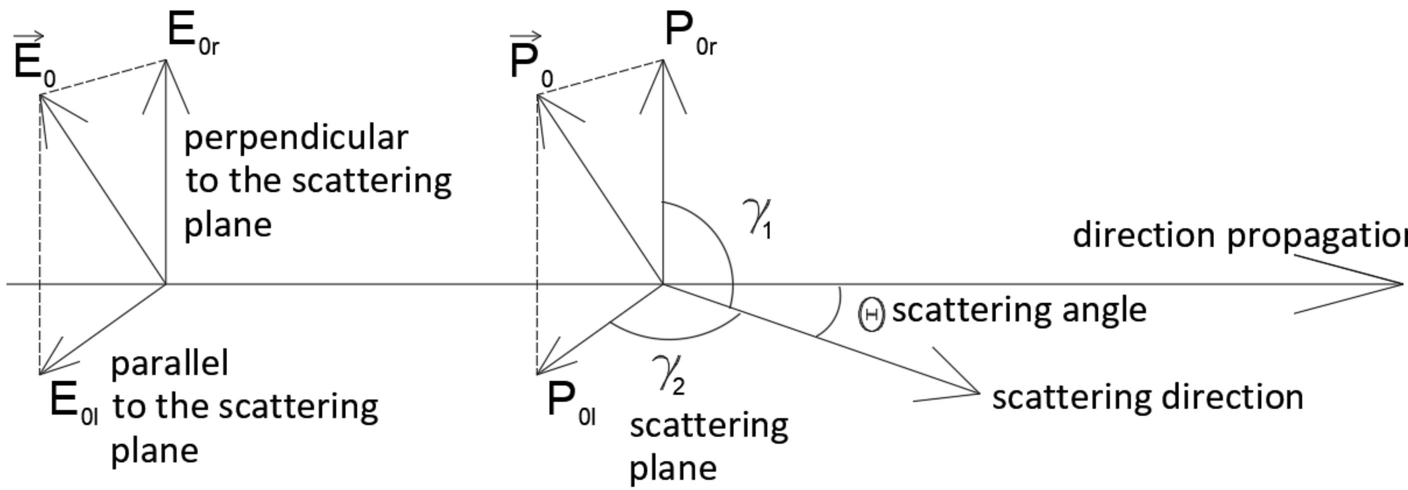
$$\frac{\partial^2\left|\vec{P}\right|}{\partial t^2} = -k^2c^2|p|$$

$$\left|\vec{E}\right| = \frac{1}{c^2}\frac{1}{r}(-k^2c^2|p|)sin\gamma$$

$$\left|\vec{E}\right| = \mathrm{E}$$

$$\mathrm{E} = -\frac{k^2}{r}\alpha E_0 e^{-ik(r-ct)} sin\gamma$$

Polarization influence (1)



Now, consider the scattering of sunlight by air molecules:

- the reference plane is defined by the directions of incident and scattered waves
- we can decompose the electric vector in into a perpendicular E_{0r} and a parallel E_{0l} (to the plane of scattering) component

γ_1, γ_2 = angles between dipoles and scattering direction

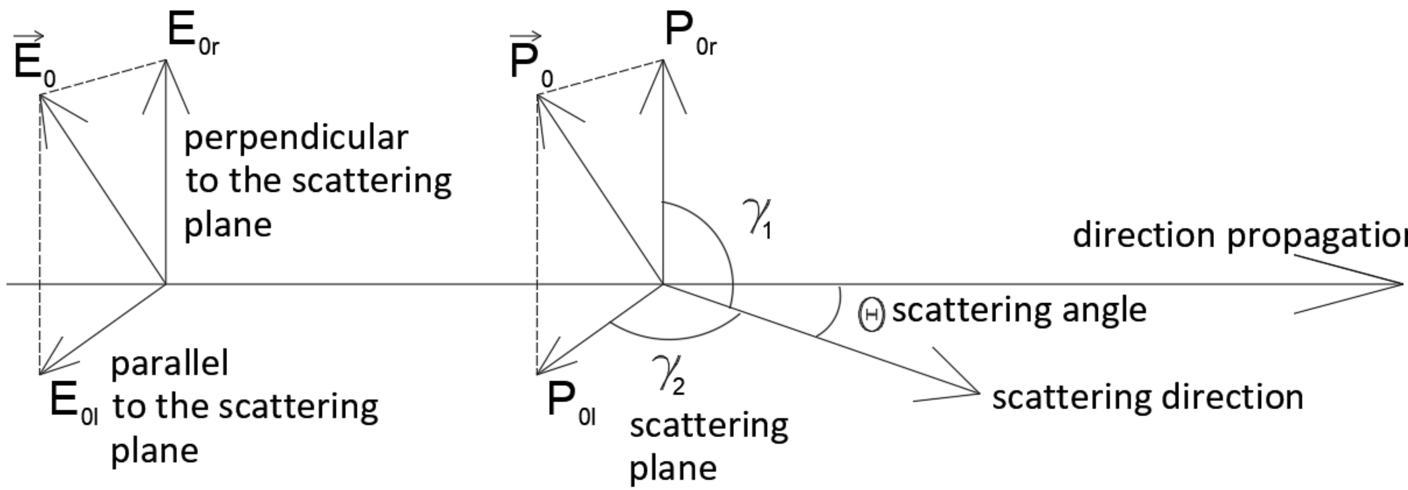
$P_{0r} = \alpha E_{0r}$

$P_{0l} = \alpha E_{0l}$

$\gamma_1 = \frac{\pi}{2}$ and $\gamma_2 = \frac{\pi}{2} - \theta$

$\gamma_1 = \frac{\pi}{2}$ always because the scattered dipole moment in the r direction is normal to the scattering plane

Polarization influence (2)



We can consider case, separately the scattering of the two electric field components by molecules assumed to be homogeneous, isotropic, spherical particles.

$$E_r = -\frac{k^2}{r}\alpha E_{0r} e^{-ik(r-ct)} sin\gamma_1 = -\frac{k^2}{r}\alpha E_{0r} e^{-ik(r-ct)}$$

$$E_l = -\frac{k^2}{r}\alpha E_{0l} e^{-ik(r-ct)} sin\gamma_2 = -\frac{k^2}{r}\alpha E_{0l} e^{-ik(r-ct)} cos\theta$$

In matrix form:

$$\begin{bmatrix} E_r \\ E_l \end{bmatrix} = -\frac{k^2}{r}\alpha e^{-ik(r-ct)} \begin{bmatrix} 1 & 0 \\ 0 & cos\theta \end{bmatrix} \begin{bmatrix} E_{0r} \\ E_{0l} \end{bmatrix}$$

Intensity (1)

We can define the intensity components (per solid angle) of the incident and scattered radiation
I = intensity
I_r= intensity on plane normal to scattering plane
I_l= intensity on the scattering plane

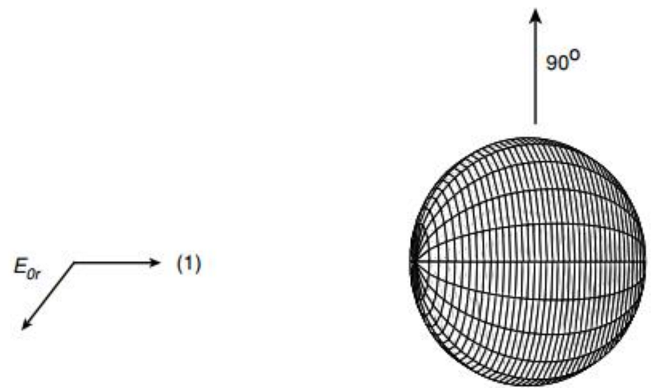
$$I = cost\left|\vec{E}\right|^2 \Rightarrow \begin{cases} I_r = cost|E_r|^2 \\ I_l = cost|E_l|^2 \end{cases} \Rightarrow \begin{cases} I_{0r} = cost|E_{0r}|^2 \\ I_{0l} = cost|E_{0l}|^2 \end{cases} \Rightarrow \begin{cases} I_r = I_{0r}\dfrac{k^4\alpha^2}{r^2} \\ I_l = I_{0l}\dfrac{k^4\alpha^2}{r^2}cos^2\theta \end{cases}$$

The scattered radiations reduced as the distance (r) squared, while the radiation with polarization perpendicular to the scattering plane is independent of the scattering angle (i.e. it is isotropic).
The radiation with polarization parallel to the scattering plane depends on the cosine squared of the scattering angle.
k is the wave number, and α is the polarizability.

Intensity (2)

If we plot the radiances on a polar diagram, we obtain :

1) Intensity in the plane perpendicular to the incident electric field



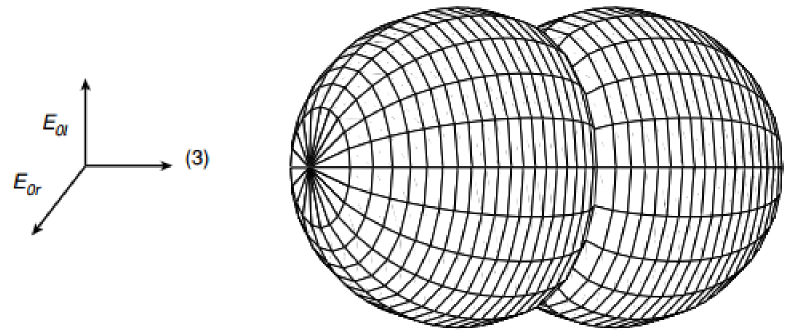
$$I_r = I_{0r}\frac{k^4\alpha^2}{r^2}$$

2) Intensity in the plane parallel to the incident electric field

$$I_l = I_{0l}\frac{k^4\alpha^2}{r^2}cos^2\theta$$

Intensity (3)

3) Non-polarized radiation (solar radiation)



$$I = I_r + I_l = \frac{k^4\alpha^2}{r^2}(I_{0r} + I_{0l}cos^2\theta)$$

Total intensity

In the case in which the incident radiation is non-polarized, as in the case of solar radiation, we have

$$I_{0l} = I_{0r} = \frac{I_0}{2}$$

and the total intensity is

$$I = \frac{k^4\alpha^2}{r^2} I_0(\frac{1 + cos^2\theta}{2})$$

Rayleigh scattering

Intensity (4)

$$I = \frac{k^4\alpha^2}{r^2} I_0 (\frac{1 + cos^2\theta}{2})$$

The electrical field oscillates in random way in every direction perpendicular to the propagation direction (electromagnetic wave).
The maximum of the intensity is in the incident direction, the minimum in the perpendicular plane.

Blue sky

The scattered intensity depends on:

- the wavelength of incident light
- the index of refraction of air molecules contained in the polarizability term.

If we look perpendicular to the direction of the Sun, we will see the darkest sky (more intense blue) of all other directions.

Thus, the intensity scattered by air molecules in a specific direction may be expressed as:

$I \propto \frac{1}{\lambda^4}$

A large portion of solar energy is contained between the blue and red regions of the visible spectrum.

Blue light $\rightarrow \lambda \approx 0.425$ μm

Red light $\rightarrow \lambda \approx 0.650$ μm

Blue light scatters about 5.5 times more intensity than red light.

Moreover, since the diffused radiance is proportional to $k^4 \propto \lambda^{-4}$, the diffused radiation at shorter wavelengths will be more intense than that at longer wavelengths.

So, the sky should have a violet color. But since solar radiance peaks around yellow, the convolution of the two trends produces the blue sky phenomenon.

Phase function (1)

To describe the angular distribution of scattered energy in conjunction with multiple scattering and radiative transfer analyses, we define a nondimensional parameter called the phase function:

Scattering distribution as a fuction of θ → P(cos θ)

$$\int_{4\pi} \frac{P(cos\theta)}{4\pi} d\Omega = 1 \text{ normalization}$$

$$\int_0^{2\pi} \int_0^{\pi} \frac{P(cos\theta)}{4\pi} sin\theta d\theta d\Omega = 1$$

RAYLEIGH SCATTERING: $P(cos\theta)$ = cost(1+$cos\theta$)

$$\frac{cost}{4\pi} \int_0^{2\pi} \int_0^{\pi} (1 + cos\theta) sin\theta d\theta d\gamma = \frac{1}{2} cost \left[\int_0^{\pi} sin\theta d\theta + \int_0^{\pi} cos\theta sin\theta d\theta \right] = 1$$

$$sin\theta d\theta = -dcos\theta \qquad cos0 = 1 \qquad cos\pi = -1$$

$$\frac{cost}{2} \left[-\int_1^{-1} dx + \int_1^{-1} x^2 dx \right] = \frac{cost}{2} \left[2 - \frac{x^3}{3} |_1^{-1} \right] = \frac{cost}{2} \left[2 + \frac{2}{3} \right] = cost \frac{4}{3} = 1$$

$$\rightarrow cost = \frac{3}{4}$$

Phase function (2)

RAYLEIGH SCATTERING: $P(cos\theta)=\frac{3}{4}\,(1+cos\theta)$

$$\frac{1+cos^2\theta}{2} = P(cos\theta)\frac{4}{3}\frac{1}{2} = P(cos\theta)\frac{2}{3}$$

We can rewrite le the Raileigh intensity as:

$$I = I_0\frac{k^4\alpha^2}{r^2}P(cos\theta)\frac{2}{3}$$

$$k^4 = \frac{(2\pi)^4}{\lambda^4} = \frac{16\pi^4}{\lambda^4}$$

$$I = I_0\frac{16\pi^4\alpha^2}{r^2}\frac{2}{3}\lambda^{-4}P(cos\theta)$$

$I \propto \frac{1}{\lambda^4}$ Blue sky

$I \propto \frac{1}{r^2}$ Intensity decreases as the square of distance

Phase function (3)

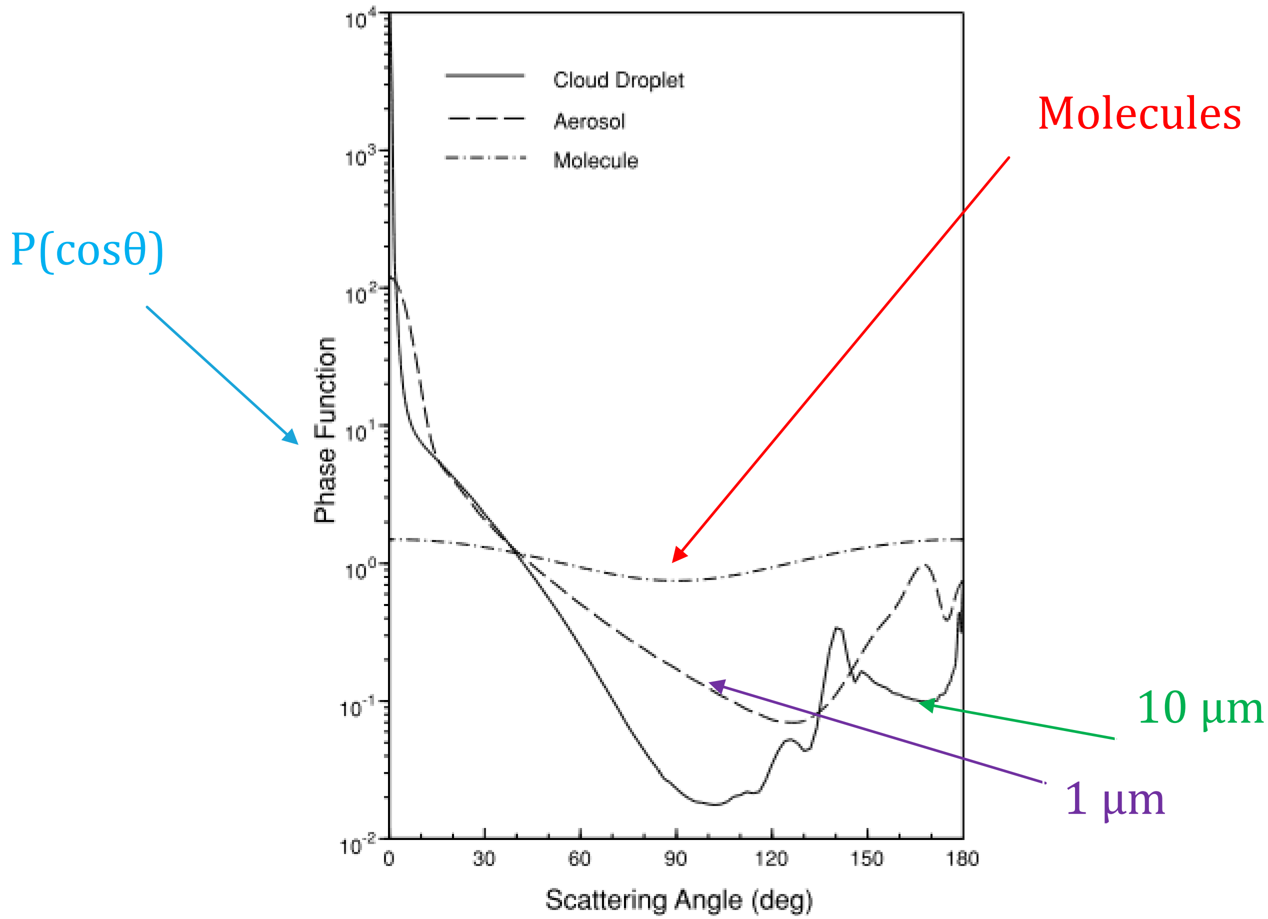


Figure 3.13 Normalized phase functions for cloud droplets (~10 μm), aerosols (~1 μm), and molecules (~10^{-4} μm) illuminated by a visible wavelength of 0.5 μm, computed from the Lorenz–Mie theory.

Scattered flux (1)

To describe the efficiency of a diffuser, the **scattering cross section** is introduced as the ratio between the irradiance spread over the entire solid angle and the incident intensity

$$\sigma_s = \frac{f_s}{F_0} \qquad [L^2] \quad \text{SCATTERING CROSS SECTION}$$

f_s = scattered flux
F_0 = incident flux density

The cross section is a sort of effective area that the diffuser presents to the incident radiation. The area is considered perpendicular to the direction of incidence and the collimated incident radiation. Therefore:

$$F_0 = I_0 \quad and \quad F_s = I_s$$

Scattered flux (2)

The flux is calculated a on a spherical surface of radius r (distance from the diffuser) which has the diffuser as its center

$\Delta\Omega_i$ = incident directions
A = area of a sphere with radius r

$$F = \int_{\Delta\Omega_i} I_x d\Omega$$

$$f_s = \int_A F_s dA = \int_\Omega F_s r^2 d\Omega =$$

$$\int_0^{2\pi}\int_0^{\pi} F_0 \frac{32}{3}\pi^4\alpha^2 \frac{1}{r^2}\frac{1}{\lambda^4} P(cos\theta) r^2 sin\theta d\theta d\varphi =$$

$$F_0 \frac{32}{3}\pi^4\alpha^2 \frac{1}{\lambda^4}\int_0^{2\pi}\int_0^{\pi} P(cos\theta) sin\theta d\theta d\varphi = F_0 \frac{32}{3}\pi^4 \frac{\alpha^2}{\lambda^4} 4\pi =$$

$$f_s = \frac{128}{3}\pi^5 \frac{\alpha^2}{\lambda^4} F_0$$

Scattering cross section

$$\sigma_s = \frac{f_s}{F_0} = \frac{128}{3}\pi^5\frac{\alpha^2}{\lambda^4}$$

Fraction of incident radiation subtracted by a unit area and scattered evenly toward all directions.

In terms of the scattering cross section, the scattered intensity can be expressed by:

$$I(\theta) = I_0\frac{\sigma_s}{4\pi r^2}P(cos\theta)$$

This is the general expression for scattered intensity, which is valid not only for molecules but also for particles whose size is larger than the incident wavelength.

$$\frac{\sigma_s}{4\pi r^2} = \frac{32}{3}\pi^4\alpha^2\frac{1}{r^2}\frac{1}{\lambda^4}$$

Polarizability

The polarizability α can be derived from the principle of the dispersion of electromagnetic waves and is given by:

$$\alpha = \frac{3}{4\pi N_s}\frac{m_\lambda - 1}{m_\lambda + 2}$$

$\alpha \propto \frac{m_\lambda - 1}{m_\lambda + 2}$

N_s= total number of molecules per unit volume
m_λ= refractive index

The polarizability depends in a complex way on the refractive index (Liou, chapter 3.3.1.2). For complex refractive indices, the real part regulates the scattering of the incident radiation, while the imaginary part regulates absorption.

Mie scattering (1)

2. Strictly speaking, **Mie scattering** is the elastic scattering from spherical particles [Mie, 1908], which includes the solution of Rayleigh scattering.

However, in LIDAR field, Mie scattering is referred to the elastic scattering from spherical particles whose size is comparable to or larger than the wavelength.

Furthermore, Mie scattering is generalized to elastic scattering from overall aerosol particles and cloud droplets, i.e., including non-spherical particles.

To precisely calculate the scattering from non –spherical particles, Mie scattering theory has to be replaced by non-spherical particle scattering theories. This is a complicated issue in the elastic LIDAR field.

Mie scattering (2)

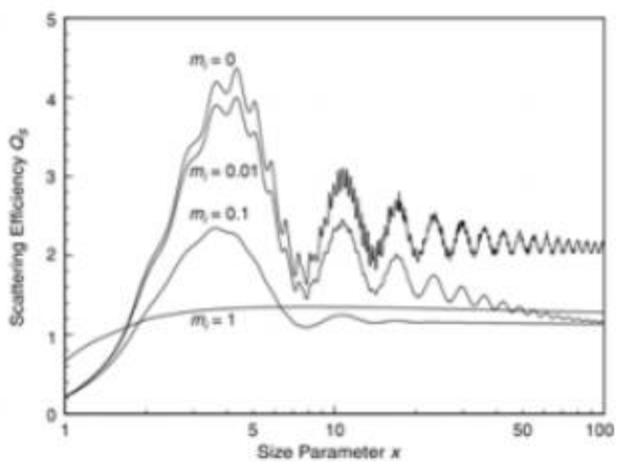
In this case, given the great variability of dimensions, instead of the cross section it is preferred to use the **scattering efficiency** parameter

$$Q_S = \frac{\sigma_S}{\pi a^2}$$

which represents the ratio between the scattering cross section and the geometric cross section. It normalizes the scattering area with the dimensions of the diffuser.

The scattering efficiency, graphed as a function of the size parameter and the imaginary part of the refractive index, has strong oscillations that fade as the value of the imaginary refractive index increases.

The oscillations depend on diffraction phenomena that are generated when the dimensions of the diffuser and the wavelength are similar. The oscillations are dampened when the absorption phenomena inside the diffuser become significant ($m_i \rightarrow 1$)



Mie scattering (3)

The phase function for the Mie scattering tends to be moved forward, a feature that grows with the size of the diffuser.

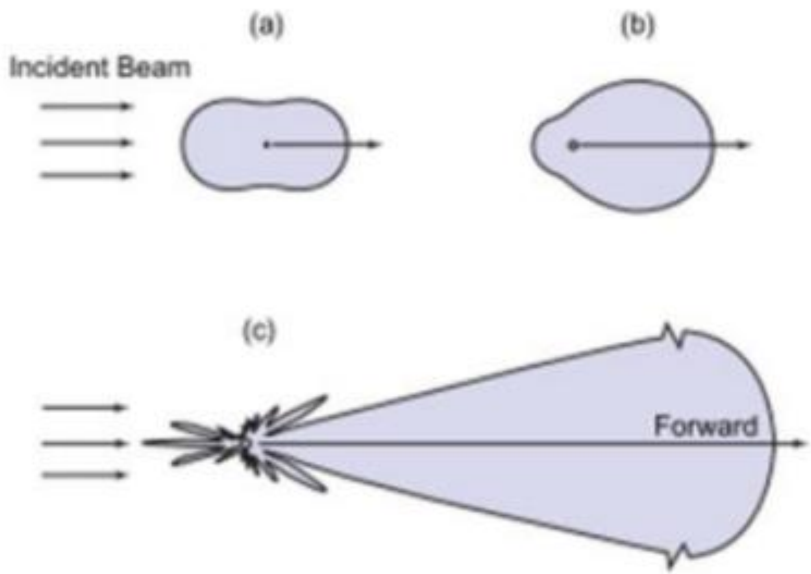


Fig. 4.12 Schematic showing the angular distribution of the radiation at visible (0.5 μm) wavelength scattered by spherical particles with radii of (a) 10^{-4} μm, (b) 0.1 μm, and (c) 1 μm. The forward scattering for the 1-μm aerosol is extremely large and is scaled for presentation purposes. [Adapted from K. N. Liou, *An Introduction to Atmospheric Radiation*, Academic Press, p. 7, Copyright (2002), with permission from Elsevier.]