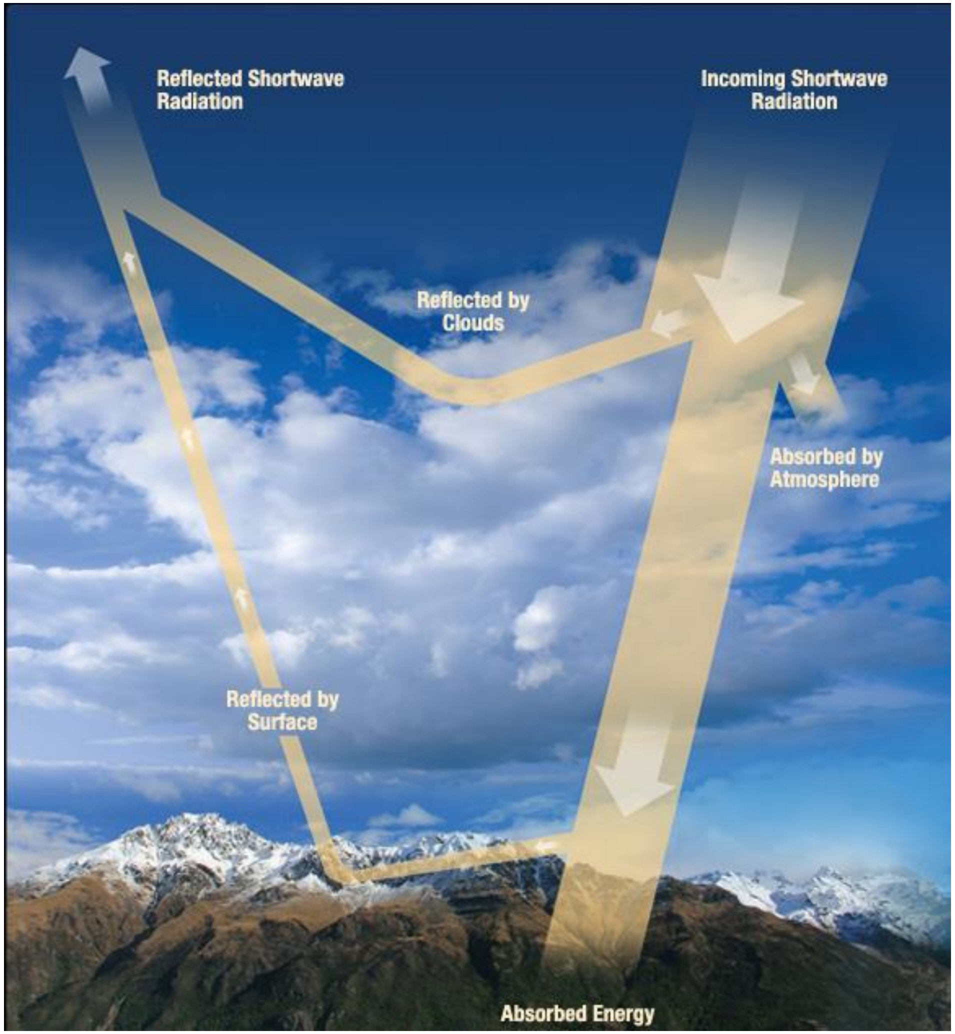
Lesson 7:
RADIATIVE TRANSFER - INTRODUCTION



Course: Laboratory of Atmospheric Remote Sensing
Laurea Magistrale in Atmospheric Science and Technology

Content

- Radiative transfer: definition, climatic effects
- Electromagnetic spectrum
- Solid angle, solid angle in hemispherical surface, solid angle in spherical surface
- Radiative variables: radiance, irradiance, flux
- Blackbody: definition, characteristics
- Blackbody radiation laws: Plank's law, Stefan-Boltzmann law, Wien's Displacement law, Kirchhoff's law

Reading material:

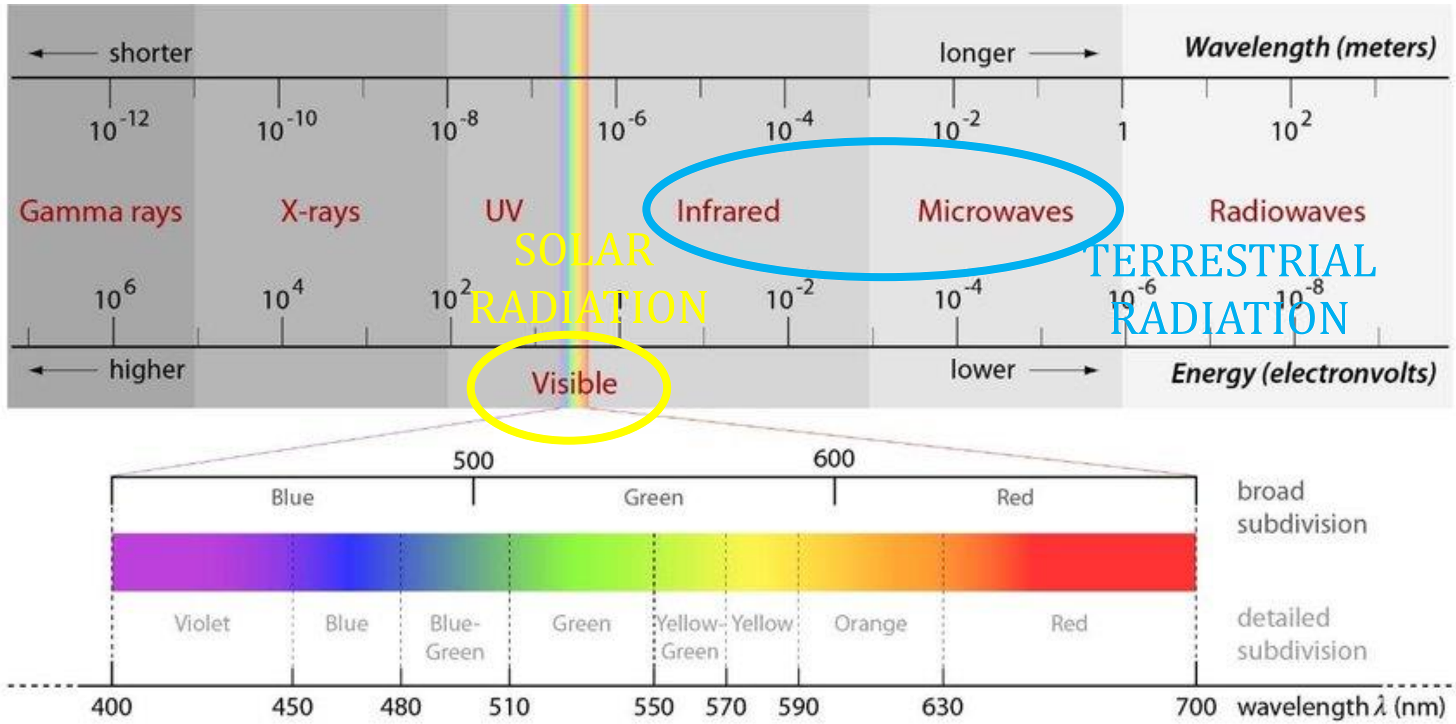
- Liou K. N., An introduction to atmospheric radiation. Chps. 1-2.
- Wallace J.M and Hobbs P.V., Atmospheric Science: An Introduction survey. Chpt. 4.

What is the radiative transfer?

Radiative transfer is the physical phenomenon of energy transfer in the form of electromagnetic radiation due to the interaction of the solar electromagnetic radiation with molecules and aerosols present in planetary atmosphere.
The propagation of radiation through a medium is affected by **absorption**, **emission**, and **scattering** processes.

The radiative transfer affects:
- Cooling/heating of the atmosphere
- Climate
- Atmosphere composition
- Remote sensing
- Atmospheric greenhouse effects
- Observation techniques

Electromagnetic spectrum



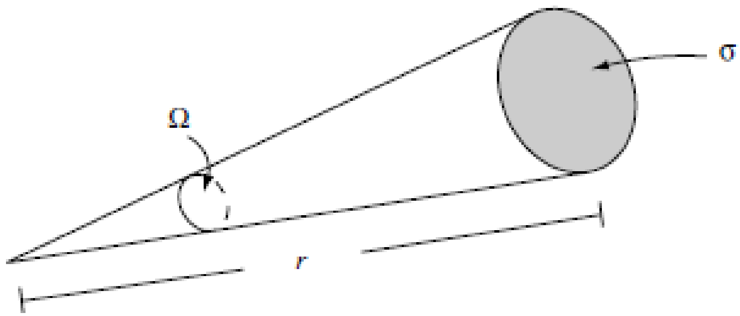
<u>DEFINITIONS</u>:

- Wavelenght λ [L] (cm, mm, µm, nm)
- Frequency $\tilde{\nu} = \frac{c}{\lambda}$ $[t]^{-1}$ (s^{-1}, Hz) c=2.998x10^8 ms^{-1}
- Wavenumber $\nu = \frac{\tilde{\nu}}{c} = \frac{1}{\lambda}$ $[L]^{-1}$ (m^{-1})
- Period $T = \frac{1}{\tilde{\nu}} = \frac{\lambda}{c}$ [t] (s)
- Angular velocity $\omega = 2\pi\tilde{\nu} = \frac{2\pi}{\lambda}c$ [rad s^{-1}]

Solid angle (1)

A solid angle Ω is defined as the ratio of the area σ of a spherical surface intercepted at the core to the square of the radius, r.

$$\Omega = \frac{\sigma}{r^2} \quad [sr]$$

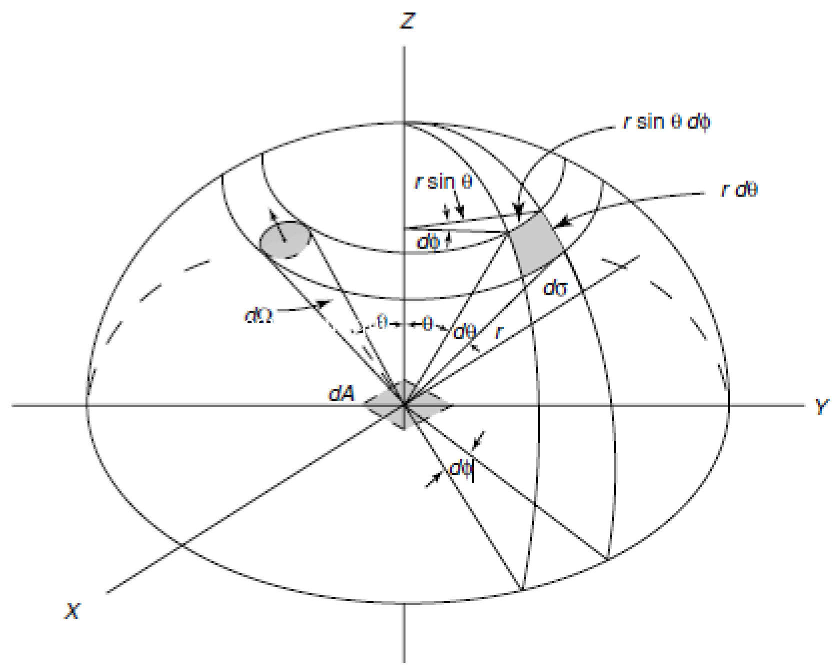


For spherical surfaces ⟶ $\Omega = 4\pi\ sr$

Differential elemental solid angle ⟶ sphere centered on point denoted O.

Solid angle (2)

Differential elemental solid angle $\longrightarrow$ sphere centered on point denoted O.



$$d\sigma = (rd\theta)(r\ sin\theta\ d\phi)$$

$$d\Omega = \frac{d\sigma}{r^2} = sin\theta\ d\theta\ d\phi$$

θ = zenith angle in polar coordinates
ϕ = azimuth angle in polar coordinates

Solid angle in hemispherical surface (S_s)

The hemispherical surface can be calculated by integrating θ from 0 to $\frac{\pi}{2}$ and ϕ from 0 to 2π

$$\theta = 0 \div \frac{\pi}{2}$$
$$\phi = 0 \div 2\pi$$

$$S_h = \int_{Em} d\sigma = \int_0^{2\pi} d\phi \int_0^{\pi/2} r^2 sin\theta d\theta = 2\pi r^2 \int_0^{\pi/2} sin\theta d\theta$$

$$sin\theta d\theta = -d(cos\theta)$$

$$S_h = -2\pi r^2 \int_1^0 d(cos\theta) = 2\pi r^2$$

$$\boxed{\Omega_h = \frac{S_h}{r^2} = 2\pi}$$

Solid angle in spherical surface (S_h)

The hemispherical surface can be calculated by integrating θ from 0 to π and ϕ from 0 to 2π

$$\theta = 0 \div \pi$$
$$\phi = 0 \div 2\pi$$

$$S_s = \int_{Sf} d\sigma = \int_0^{2\pi} d\phi \int_0^{\pi} r^2 sin\theta d\theta = 2\pi r^2 \int_0^{\pi} sin\theta d\theta$$

$$sin\theta d\theta = -d(cos\theta)$$

$$S_s = -2\pi r^2 \int_1^{-1} d(cos\theta) = 4\pi r^2$$

$$\boxed{\Omega_s = \frac{S_S}{r^2} = 4\pi}$$

Radiative variables

Let's introduce some radiative variables of atmospheric interest. All variables are dependent on the wavelength and therefore are defined per unit of spectral interval $[L]^{-1}$. To indicate this prerogative, the adjective "spectral" is added to their name and the inverse of a unit of length (μm^{-1}, cm^{-1}) to the unit of measurement. Sometimes the dependence in wavelength is transformed into wave number or frequency and consequently the units of measurement are changed.
In the definitions and use we consider them integrated over the entire electromagnetic spectrum and therefore independent of the wavelength, unless otherwise specified.

Radiative energy E_λ [J]

Time interval dt [s]

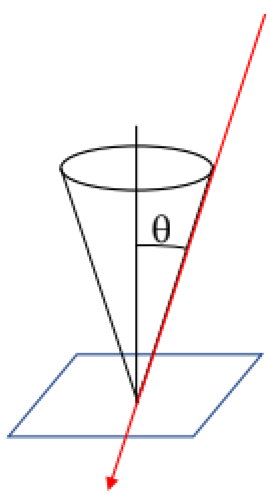
Area dA [m^2]

Radiative variables: Radiance

<u>Radiance or intensity (I)</u> ($Wm^{-2}sr^{-1}$)
The amount of energy that
1. crosses an infinitesimal area from $dA_{\perp}$ perpendicular to the direction of propagation
2. coming from directions contained within a solid angle $d\Omega$ around the direction $\Omega = (\theta, \phi)$
3. in a time dt
It will be proportional to $d^2E = I(\theta, \phi, t)dA_{\perp}d\Omega dt$

$I(\theta, \phi, t)$ is called *radiance* or *intensity* and is equal to radiant energy that crosses a unit area perpendicular to the direction $I(\theta, \phi)$ in the unit of time per unit of solid angle.

$$I(\theta, \phi, t) = \frac{d^2E}{dA_{\perp}d\Omega dt}$$



The orientation of the surface with respect to the direction of propagation depends only on the zenith angle and the relationship between the effective area and the perpendicular portion is

$$dA_{\perp} = cos\theta dA$$

Radiative variables: Radiance

So

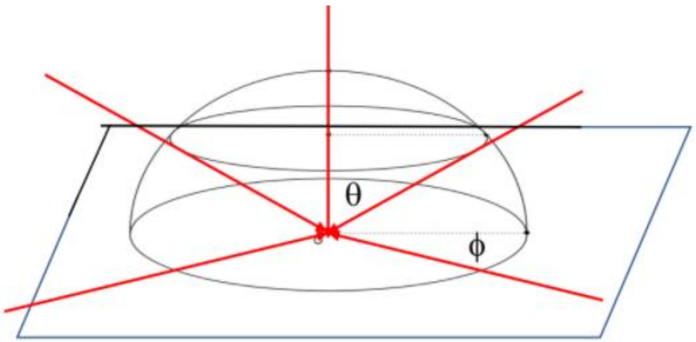
$$I(\theta, \phi, t) = \frac{d^2 E}{cos\theta dA\, d\Omega dt}$$

The dimensions of the radiance are $[E][L]^{-2}[t]^{-1}$. The units of measurement $[E][t]^{-1}$ are collected in a power; the unit of solid angle (which would be dimensionless) is explicitly indicated. Hence the radiance is expressed in (W m^{-2} sr^{-1}).

The spectral radiance (I_λ, radiance per unit of wavelength interval) is expressed in (W m^{-2} sr^{-1} μm^{-1}). A sub-multiple of the meter is used to avoid having to deal with very small values.

Radiative variables: Irradiance

Irradiance or flux density (W m^{-2})
Radiant energy that passes in the unit of time through a unit area coming from all directions of a hemisphere. It is obtained by integrating the radiance over a solid angle of 2π.
The contribution of the radiance coming from a zenith angle θ must be weighted by cosθ to take into account the inclination with respect to the surface, and multiplied by the angle solid dΩ.



$$\mathrm{dF(t)} = \mathrm{I}(\theta, \phi, t) cos\theta d\Omega$$
$$= \mathrm{I}(\theta, \phi, t) cos\theta sin\theta d\theta d\phi$$

$$F(t) = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} I(\vartheta, \psi, t)\, cos\theta sin\theta d\theta d\phi$$

If the radiance is isotropic, that is, it does not depend on the direction of origin:

$$F(t) = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} I(t)\, cos\theta sin\theta d\theta d\phi = 2\pi I(t) \int_0^{\frac{\pi}{2}} cos\theta sin\theta d\theta$$
$$= 2\pi I(t) \int_0^{1} sin\theta d\theta = \pi I(t)$$

Radiative variables: Irradiance

If the radiance is collimated, i.e. all the rays come from the same direction (θ_0,ϕ_0) as can be approximated the case of solar radiation that affects the Earth.

$$I(\theta,\psi,t) = I_{\odot}(t)\delta(\theta-\theta_0)\delta(\phi-\phi_0)$$

In this case (θ_0,ϕ_0) are the zenith and azimuth angles of the sun, δ is the Dirac delta and $I_{\odot}(t)$ is the solar radiance incident perpendicular to the earth's surface at time t.

$$F(t)_{\odot} = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} I_{\odot}(t)\delta(\theta-\theta_0)\delta(\phi-\phi_0)\ cos\theta sin\theta d\theta d\phi$$

$$= I_{\odot}(t)\int_0^{\frac{\pi}{2}}\delta(\theta-\theta_0) cos\theta sin\theta d\theta \int_0^{2\pi}\delta(\phi-\phi_0)\ d\phi$$

Radiative variables: Irradiance

$$F(t)_{\odot} = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} I_{\odot}(t)\delta(\theta - \theta_0)\delta(\phi - \phi_0)\ cos\theta sin\theta d\theta d\phi$$
$$= I_{\odot}(t) \int_0^{\frac{\pi}{2}} \delta(\theta - \theta_0) cos\theta sin\theta d\theta \int_0^{2\pi} \delta(\phi - \phi_0)\ d\phi$$

The two integrals are notable integrals that yield the results

$$\int_0^{\frac{\pi}{2}} \delta(\theta - \theta_0) cos\theta sin\theta d\theta = cos\theta_0$$
$$\int_0^{2\pi} \delta(\phi - \phi_0)\ d\phi = 1$$

So the solar irradiance (collimated radiation) is equal to the solar radiation multiplied for the cosine of the solar zenith angle ($cos\theta_0$); often the cosine of the zenith angle is indicated with μ_0.

$$F(t)_{\odot} = I_{\odot}(t) cos\theta_0 = \mu_0 I_{\odot}(t)$$

Radiative variables: Flux

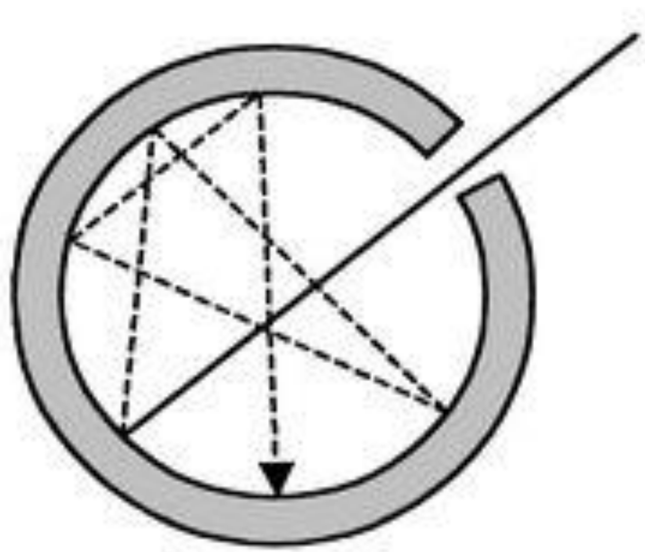
Radiant energy that passes in the unit of time through area A coming from all directions of a hemisphere. It is obtained by integrating the irradiance on area A

$$f(t) = \int_A F(t)dA$$

Blackbody

<u>Configuration of material with complete absorption and maximum emission at a certain temperature.</u>

Can be described as a cavity with a small entrance hole in which most of the radiant flux entering this hole from the outside will be trapped within the cavity, regardless of the material and surface characteristics of the wall. The probability that any of the entering flux will escape back through the hole is so small that the interior appears dark.



Blackbody radiation laws

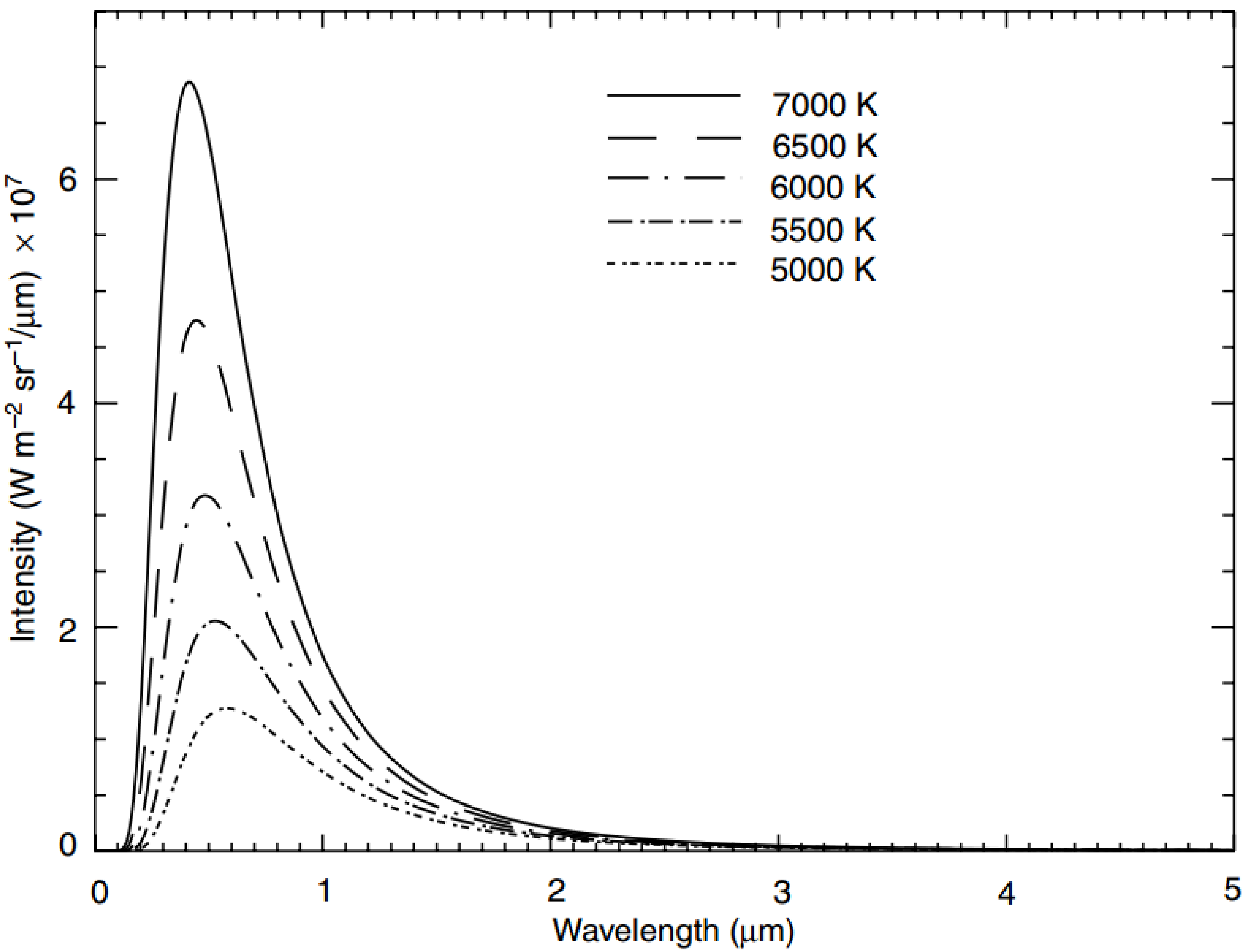
1) Plank's law

The Planck function relates the emitted monochromatic intensity to the frequency and the temperature of the emitting substance.
The blackbody radiant intensity increases with temperature, while the wavelength of the maximum intensity decreases with increasing temperature.
The spectral radiance emitted by a black body is isotropic (does not depend on the direction) and is expressed by Plank's law:

$$\boxed{B_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{hc/K\lambda T} - 1)} = \frac{C_1\lambda^{-5}}{\pi(e^{C_2/\lambda T} - 1)}}$$

T = absolute temperature
c = speed of light
h = Plank constant = 6,626 x 10^{-34} J sec
K = Boltzmann constant = 1,3806 x 10^{-23} J deg^{-1}
$C_1 = 2\pi hc^2$
$C_2 = hc/\mathrm{K}$

Blackbody radiation laws



Blackbody radiation laws

2) Stefan-Boltzmann law

The total radiant intensity of a blackbody can be derived by integrating the Planck function over the entire wavelength domain from 0 to ∞.

The flux density emitted by a blackbody is proportional to the fourth power of the absolute temperature.

$$B(T) = \int_0^\infty B_\lambda(T)\, d\lambda = \int_0^\infty \frac{2hc^2\lambda^{-5}}{(e^{hc/K\lambda T} - 1)}\, d\lambda = \sigma T^4$$

where

$$\sigma = \frac{2\pi^4 K^4}{15c^3h^3}$$

Since blackbody radiation is isotropic, the flux density emitted by a blackbody is:

$$\boxed{F = \pi B(T) = \sigma T^4}$$

where

σ = Stefan-Boltzmann constant = 5.67x 10−8 J $m^{-2}sec^{-1}\,deg^{-4}$

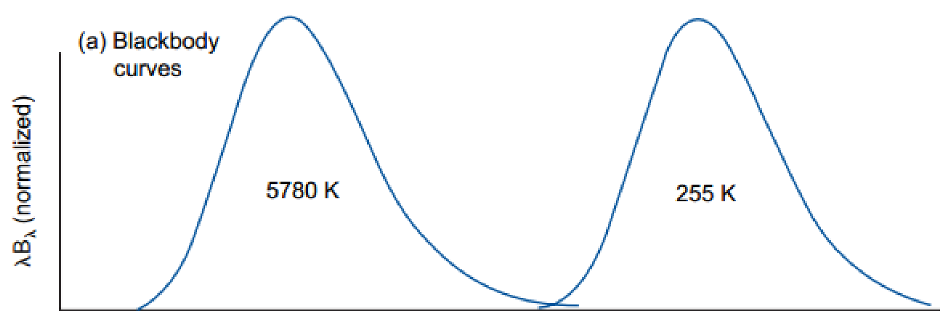
Blackbody radiation laws

3) Wien's Displacement Law

The wavelength of the maximum intensity of blackbody radiation is inversely proportional to the temperature.
From this relationship, we can determine the temperature of a blackbody from the measurement of the maximum monochromatic intensity. The wavelength at which the black body emits at maximum radiance is obtained by canceling the derivative of the Plank function. The solution is called Wien's Law:

$$\frac{\partial B_{\lambda}(T)}{\partial \lambda} = 0 \quad \rightarrow \quad \boxed{\lambda_{max}(T) = \frac{a}{T}}$$

a = 2.897 x 10^{-3} m K^{-1}



Since the Sun and the Earth are at very different temperatures, the emission spectra are almost completely separate.

Blackbody radiation laws

4) Kirchhoff's Law

Since the blackbody absorbs the maximum possible radiation, it has to emit that same amount of radiation. If it emitted more, equilibrium would not be possible, and this would violate the second law of thermodynamics.

The emissivity of a given wavelength, ε_λ (defined as the ratio of the emitting intensity to the Planck function), of a medium is equal to the absorptivity, A_λ (defined as the ratio of the absorbed intensity to the Planck function), of that medium under thermodynamic equilibrium.

$$\varepsilon_\lambda = A_\lambda$$

For a blackbody absorption is a maximum and so is emission:

$$\boxed{\varepsilon_\lambda = A_\lambda = 1}$$

For a grey body:

$$A_\lambda = \varepsilon_\lambda < 1$$