Lesson 1: Planetary Boundary Layer

Course: Laboratory of Atmospheric Remote Sensing
Laurea Magistrale in Atmospheric Science and Technology
Planetary Boundary Layer: definition and main characteristics, mean wind, waves and turbulence

Turbulence: mechanical and thermal turbulence, buoyancy, adiabatic and environmental lapse rate

Atmospheric stability

Variables of interest: Reynolds number, Reynolds decomposition, standard deviation of fluctuating velocity, Taylor’s hypothesis, convective velocity scale, friction velocity, Monin-Obukhov length

Monin-Obukhov similarity theory

PBL depth over oceans, low and high pressure regions

PBL vertical structure and daily evolution

Convective PBL: main characteristics, height and turbulent velocities

Stable PBL: main characteristics, height and turbulent velocities
Micrometeorology
Urban boundary layer: definition, vertical structure, aerodynamic roughness length, zero displacement height
Flow around an isolated building
Flow regime in 2D and 3D street canyons
Dispersion and stability

Reading material:
R.B. Stull “An Introduction to Boundary Layer Meteorology”
Chapter 1
PLANETARY BOUNDARY LAYER
The Atmospheric Boundary Layer (ABL), or Planetary Boundary Layer (PBL), is the lowest 1-3 km of the atmosphere, where the transport processes modify the structure and the composition of the atmosphere.

The remnant of the atmosphere above is termed “free” atmosphere.

The PBL is defined as the part of the troposphere directly influenced by the presence of the earth’s surface and responds to surface forcings with a timescale of about an hour or less and with typical spatial scales of a few kilometers.

The main forcings are:
• heat transfer from/to the ground
• evapotranspiration
• frictional drag
• pollutant emissions
• flow modification due to topography
Day time boundary layer is usually very turbulent, due to ground-level heating.
## COMPARISON OF BOUNDARY LAYER AND THE FREE ATMOSPHERE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Property</th>
<th>Boundary Layer</th>
<th>Free Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulence</td>
<td>• Almost continuously turbulent over its whole depth.</td>
<td>• Turbulence in convective clouds, and sporadic CAT in thin layers of large horizontal extent.</td>
</tr>
<tr>
<td>Friction</td>
<td>• Strong drag against the earth's surface. Large energy dissipation.</td>
<td>• Small viscous dissipation.</td>
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<tr>
<td>Dispersion</td>
<td>• Rapid turbulent mixing in the vertical and horizontal.</td>
<td>• Small molecular diffusion. Often rapid horizontal transport by mean wind.</td>
</tr>
<tr>
<td>Winds</td>
<td>• Near logarithmic wind speed profile in the surface layer. Subgeostrophic, cross-isobaric flow common.</td>
<td>• Winds nearly geostrophic.</td>
</tr>
<tr>
<td>Vertical Transport</td>
<td>• Turbulence dominates.</td>
<td>• Mean wind and cumulus-scale dominate</td>
</tr>
<tr>
<td>Thickness</td>
<td>• Varies between 100 m to 3 km in time and space. Diurnal oscillations over land.</td>
<td>• Less variable. 8-18 km. Slow time variations.</td>
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</table>
Temporal trends in the lower troposphere obtained by rawinsonde (wind speed and direction) soundings made every several hours in Oklahoma.

Near the ground the net diurnal variation is due to the absorption of solar radiation by the ground, that warms and cools in response of the radiation, which in turn forces changes in the UBL via transport processes.

In the free atmosphere the diurnal cycle of air temperature is not evident.
Three categories of air flow can exist in the UBL:
- **Mean wind** (horizontal: 2-10 m/s, vertical: 0.01-0.1 m/s) is mainly responsible for the horizontal transport, or *advection*, of pollutants, heat and momentum. It tends to zero near the ground because of the friction with surface and obstacles.
- **Waves** can be generated by mean-wind shears, flow over obstacles and can propagate also for long distances in the case of thunderstorms or explosions. Typically transport little heat, humidity and scalars during nighttime.
- **Turbulence**, consists of several irregular superimposed swirls of motion with different size (1 mm - 3000 m) called *eddies*. The relative strengths of these scale eddies define the *turbulence spectrum*.

Turbulence can be generated by the friction with the ground:

- **Solar heating**: during sunny days causes thermals and fover the ascending motion of warm/cold air
- **Frictional drag**: at the contact with the surface
- **Obstacles**: like trees or buildings can perturb the flow and generate turbulence near the obstacles themselves
TURBULENCE

- Mechanical turbulence (wind-related)
- Thermal turbulence (temperature-related)

- Eddy – gusts, swirls of wind in the vertical plane caused by turbulence
- Carry heat, momentum, water vapor, CO₂ etc
- As large as the boundary layer, as small as a few molecules.
MECHANICAL AND THERMAL TURBULENCE

MECHANICAL TURBULENCE
- Shear stress
- Directional shear
- Surface frictional effects

THERMAL TURBULENCE
- Density depends on temperature (ideal gas law)

\[ \text{PV} = nRT \quad \rightarrow \quad P = \rho RT \quad \rightarrow \quad \rho = \frac{P}{RT} \]
BUOYANCY IN FLUIDS

Parcel colder than environment

Parcel warmer than environment
**ADIABATIC AND ENVIRONMENTAL LAPSE RATE**

**Adiabatic lapse rate:**
Rate of temperature change that an air parcel experiences as it changes elevation without any heat exchange.

\[
\left( \frac{dT}{dz} \right)_{\text{adiab}} = \Gamma = -\frac{g}{c_P}
\]

**Environmental lapse rate:**
Actual rate of temperature change with height of the current atmosphere.

\[
\left( \frac{dT}{dz} \right)_{\text{env}}
\]
1. **Unstable conditions**: usually daytime, caused by swirling eddied rising off the heated surface because they are more buoyant than surrounding air.

\[
\left( \frac{dT}{dz} \right)_{env} < \Gamma
\]

2. **Stable conditions**: usually at night, only mean wind and waves, little turbulence causes only horizontal transport. Stable air provides excellent conditions for high pollution levels.

\[
\left( \frac{dT}{dz} \right)_{env} > \Gamma
\]

3. **Neutral conditions**: upper PBL at night, turbulence at equal intensity in all directions.

\[
\left( \frac{dT}{dz} \right)_{env} = \Gamma
\]
Examples of stability and instability in relation to air and parcel temperatures (created by Britt Seifert).
ATMOSPHERIC STABILITY (3)

The effects of moisture change the lapse rate of the air parcel and, therefore, affects stability.

**Absolutely stable**: the environmental lapse rate is less than the moist adiabatic lapse rate. This means that a rising air parcel will always cool at a faster rate than the environment, even after it reaches saturation. If an air parcel is cooler at all levels, then it will not be able to rise, even after it becomes saturated (when latent heating will counteract some cooling).

A thermodynamic diagram showing the stability of the atmosphere based on the dry ($\Gamma_d = 9.8 \text{ K km}^{-1}$) and moist ($\Gamma_m = 4.5 \text{ K km}^{-1}$) adiabatic lapse rates (Created by Britt Seifert).
Absolutely unstable: the environmental lapse rate is greater than the dry adiabatic lapse rate. This means that a rising air parcel will always cool at a slower rate than the environment, even when it is unsaturated. This means that it will be warmer (and less dense) than the environment, and allowed to rise.

A thermodynamic diagram showing the stability of the atmosphere based on the dry ($\Gamma_d = 9.8 \text{ K km}^{-1}$) and moist ($\Gamma_m = 4.5 \text{ K km}^{-1}$) adiabatic lapse rates (Created by Britt Seifert).
Conditionally unstable: the environmental lapse rate is between the moist and dry adiabatic lapse rates. This means that the buoyancy of an air parcel depends on whether or not it is saturated.

In a conditionally unstable atmosphere, an air parcel will resist vertical motion when it is unsaturated, because it will cool faster than the environment at the dry adiabatic lapse rate. If it is forced to rise and is able to become saturated, however, it will cool at the moist adiabatic lapse rate.
Turbulent flows occur when the inertial forces acting on the fluid are much greater than the stabilizing viscous forces. The *Reynolds number* is a measure of the ratio of these two forces.

It is defined as:

\[
Re = \frac{Ud}{\nu}
\]

where
- \( U \) is the mean velocity of the flow
- \( d \) is the length scale of the flow (e.g., the diameter of the pipe through which the fluid is flowing)
- \( \nu \) is the kinematic viscosity of the fluid (for air \( \nu=5\times10^{-6} \text{ m}^2/\text{s} \))

Turbulent flows are characterized by Reynolds numbers much greater than 1000.
Assumption: the flow is *steady*, in the sense that time averages converge to specific values when the averaging time is long enough.

We can write the instantaneous velocity, $u(t)$, using the so-called *Reynolds decomposition* as follows:

$$u(t) = U + u'(t)$$

where $U$ is the time average or mean defined by

$$U = \lim_{T \to \infty} \frac{1}{T} \int_0^T u(t) \, dt$$

and $u'(t)$ is the fluctuating component.

Moreover, we can define $u$ and $v$ as the horizontal components of the velocity are denoted by and $w$ as the vertical wind component.
The statistic of greatest relevance to dispersion of pollution is the **standard deviation of the fluctuating velocity** defined by

\[
\sigma_U = \left[ \lim_{T \to \infty} \frac{1}{T} \int_0^T u^2 dt \right]^{0.5}
\]

Notice that, by definition, the time averages of the fluctuating quantities are zero. However, the average of the product of two fluctuating quantities is not zero.
We would like to be able to take snapshots of the eddies in three dimensions and measure all their sizes each instant. We can only measure the fluctuations of a variable such as wind speed, specific humidity, or temperature with a sensor at one location for a period of time. In this way, we watch the eddies drift by the sensor. But the eddies could be changing size and shape as they drift by the sensor.

In 1938, Taylor suggested that in some cases the turbulent eddies (that we can think of as large air parcels) can be considered frozen: the measurements of wind speed can be used to translate the turbulence as a function of time to the correspondent function of space.

The turbulence is not really frozen, it is only an approximation of the reality.
EDDIES PERIOD

\[ M^2 = U^2 + V^2 \]

\[ P = \frac{\lambda}{M} \]

M = total wind speed
U = eastward Cartesian wind component
V = northward Cartesian wind component
\( \lambda \) = eddy diameter
P = period to pass by a stationary sensor
TEMPERATURE APPROXIMATION

The temperature variation with time at any location (such as where a sensor might be placed) is the sum of the total derivative and the temperature advection:

\[
\frac{\partial T}{\partial t} = \frac{DT}{dT} - \vec{U} \cdot \nabla T
\]

The total derivative is the change in temperature of an air parcel moving past the sensor.

The temperature advection is the change in temperature at the sensor due to the advection of warmer or colder air past the sensor.

If we consider the Taylor’s hypothesis, the turbulence is frozen and, so, the change in temperature within each eddy is negligible.

\[
\frac{DT}{dT} = 0
\]

and so

\[
\frac{\partial T}{\partial t} = -\vec{U} \cdot \nabla T
\]
TEMPERATURE APPROXIMATION

\[ \frac{\partial T}{\partial t} = -\vec{U} \cdot \vec{\nabla} T \]

Local temperature gradients, which might be present from one side of an eddy to another, are advected across the sensor by the mean wind without the eddy changing.

- When is this condition valid?
  - Experiments suggest that this hypothesis is valid when the variation of the wind speed due to turbulence is less than \( \frac{1}{2} \) of the mean wind speed.
CONVECTIVE VELOCITY SCALE (1)

Heat flux from the ground to the atmosphere creates buoyancy forces, which in turn generate turbulent velocities. We can estimate these *turbulent velocities* by considering an air parcel of unit mass with a temperature excess of $\Delta T$ over its surroundings that it acquires at the heated ground.

Considering the specific volume (volume per unit mass) of the parcel as $v$, the two forces on the parcel (neglecting drag forces for the moment) are:

- Downward gravitational force = $g$
- Upward buoyancy force = $vg\rho_s$;

where $\rho_s$ is the density of the surroundings.
The net upward force is:

\[ F_u = \nu g \rho_s - g = g(\nu \rho_s - 1) \]

\[ \nu = \frac{1}{\rho} \]

\[ F_u = g \left( \frac{\rho_s}{\rho} - 1 \right) = g \left( \frac{T}{T_s} - 1 \right) \quad \text{(from the gas law)} \]

\[ F_u = g \frac{\Delta T}{T_s} \approx g \frac{\Delta \theta}{\theta_s} \]

This force, acting over a distance \( z \), generates a kinetic energy \( \approx w^2 \) so that

\[ g \frac{\Delta \theta z}{\theta_s} \approx w^2 \]
CONVECTIVE VELOCITY SCALE (3)

Now let us multiply both sides of the equation by $w$

$$g \frac{(\Delta \theta w)z}{\theta_s} \approx w^3$$

The term inside the parentheses is the velocity of the parcel multiplied by the temperature excess carried by the parcel.

This quantity is proportional to the surface heat flux:

$$\Delta \theta w \sim \frac{H}{\rho c_p}$$

Then

$$w \approx \left( \frac{g}{\theta_s \rho c_p} \right)^{1/3}$$

$$\frac{H}{\rho c_p}$$ is referred to as the **kinematic heat flux**, and is denoted by

$$\frac{H}{\rho c_p} = Q_0$$
Now, define a free convection scale, $u_f$:

$$u_f = \left(\frac{g}{T_0} Q_0 z\right)^{1/3}$$

where $T_0$ is the near surface temperature which is approximately equal to $\theta_s$ in a well-mixed convective boundary layer.

Another velocity scale that is used to characterize a boundary layer dominated by surface heating is the *convective velocity scale* given by:

$$w = \left(\frac{g}{T_0} Q_0 z_i\right)^{1/3}$$

where $z_i$ is the PBL height, which is also called the *mixed layer* because vertical motion induced by buoyancy leads to vigorous vertical mixing of the properties of the boundary layer.
FRICTION VELOCITY (1)

Except very close to the ground, the horizontal shear stress is supported by macroscopic turbulent motion. When parcels of air travel vertically, they exchange momentum between layers of air with different velocities. Vertical gradients in horizontal mean velocity lead to changes in instantaneous horizontal velocities during this transfer of momentum.

If we denote the horizontal velocity fluctuation, \( u' \), created by a parcel of air with vertical velocity, \( w \), the horizontal momentum transferred across a horizontal layer by the parcel is \( \rho u'w' \), where \( \rho \) is the air density. If the horizontal shear stress is roughly constant with height and is equal to the surface stress, \( \tau_0 \), then

\[
\tau_0 = -\overline{\rho u'w'}
\]

where the overbar denotes a time average. The negative sign ensures that \( \tau_0 \) is positive because a positive \( w \) is associated with a negative \( u \) when the mean horizontal velocity increases with height.
These arguments suggest that the turbulent velocities associated with shear production of turbulence scale with the surface friction velocity, \( u \), defined by

\[
 u_* = \sqrt{\frac{\tau_0}{\rho}}
\]

Buoyant and shear production of turbulence operate together to determine the structure of the boundary layer.

A length scale, referred to as the *Monin-Obukhov length*, allows us to combine the effects of these mechanisms into a single framework that describes the vertical structure of the near surface atmospheric boundary layer.
The absolute value of the *Monin-Obukhov length*, $L$, is roughly the height at which the turbulent velocity generated by shear is equal to that produced by buoyancy:

$$u_* \sim u_f(z = L) = \left(\frac{g}{T_0} Q_0 L\right)^{1/3}$$

which yields the definition

$$L = -\frac{T_0 u_*^3}{gkQ_0}$$

where $k$ is the von Karman constant 0.4. The negative sign indicates that when $Q_0$ is positive during the day, $L$ is negative and positive when the heat flux is toward the ground. So $L$ is positive when the boundary layer is stable, and negative when it is unstable.

Shear production of turbulence dominates that by buoyancy at heights below the Monin-Obukhov length, while buoyant production becomes dominant above it.
The Monin-Obukhov similarity theory (MOST) permits to investigate the mean and the turbulent structure of the boundary layer at heights below the order of magnitude of the Monin-Obukhov length.

The mean temperature and velocity gradients can be represented by universal functions if the velocity, temperature, and height are scaled appropriately. The velocity scale is $u_*$, the height scale is $L$.

The temperature scale, $\theta_*$, is given by

$$\theta_* = -\frac{Q_0}{u_*}$$
MONIN-OBUKHOV SIMILARITY THEORY (2)

Let us consider a neutral boundary layer, that is dominated by shear. In such a boundary layer, the mean velocity gradient is of the same order as the velocity gradient across the dominant turbulent eddy at that height. We assume that the dominant eddy at a height \( z \) has a length scale of order \( z \) and a velocity scale of order \( u_* \).

Then, measurements indicate that we can write

\[
\frac{dU}{dz} = \frac{u_*}{kz}
\]

where \( k=0.4 \) is the von Karman constant. Integration yields the logarithmic expression for the mean wind speed at height \( z \):

\[
U(z) = \frac{u_*}{k} \ln \left( \frac{z}{z_0} \right)
\]

where \( z_0 \) is the roughness length, related to the physical dimensions of the objects at the surface.
Monin-Obukhov Similarity theory states that we can account for the effects of heat flux by modifying the neutral-conditions log-law as follows:

\[
\frac{dU}{dz} = \frac{u_*}{kz} \phi_m \left( \frac{Z}{L} \right)
\]

and the potential temperature gradient can be expressed as

\[
\frac{d\theta}{dz} = \frac{\theta_*}{kz} \phi_h \left( \frac{Z}{L} \right)
\]

The surface heat flux goes to zero, \( L \to \infty \) and \( z/L \to 0 \). This means that \( \phi_m(0) = 1 \) and \( \phi_h(0) = 1 \) to be consistent with the gradient in the neutral boundary layer. \( \theta_* \) goes to zero when the surface heat flux goes to zero.
The last formulations are supported by observations which indicate that

\[ \phi_m = (1 - 15 \frac{Z}{L})^{-1/4} \quad \text{for } L < 0 \]

\[ \phi_m = 1 + 4.7 \frac{Z}{L} \quad \text{for } L > 0 \]

and

\[ \phi_h = 0.74(1 - 9 \frac{Z}{L})^{-1/2} \quad \text{for } L < 0 \]

\[ \phi_h = 0.74 + 4.7 \frac{Z}{L} \quad \text{for } L > 0 \]
PBL DEPTH OVER OCEANS

PBL height varies slowly in space and time.

- Little diurnal cycle in sea surface temperature because of mixing at the top of the ocean
- Large heat capacity of water: water can absorb large amounts of heat with small temperature change

PBL height changes mainly because of synoptic and mesoscale processes of vertical motion and advection of different air masses over the sea surface.

Very gentle variation: PBH height varies of about 10% over a horizontal distance of 1000 km.
Expections: borders between ocean currents with different temperatures
The upward motions carry boundary-layer air away from the ground to large altitudes throughout the troposphere. It is complex to identify a PBL top so, cloud base is typically considered as an arbitrary cut-off of the PBL.
PBL STRUCTURE OVER HIGH PRESSURE REGIONS

- Cloud Layer
- Entrainment Zone
- Capping Inversion
- Convective Mixed Layer
- Residual Layer
- Stable (Nocturnal) Boundary Layer
- Surface Layer
- Night
- Sunset
- Midnight
- Sunrise
- Noon
**Mixed Layer (Convective Boundary Layer):**
- turbulence driven by convection (large eddies or thermals)
- heat transfer from solar heating of the ground to the atmosphere
- mixed layer grows by entrainment of air from above it
- virtual temperature nearly adiabatic in middle; superadiabatic (i.e., potential temperature decreases with height) near surface; subadiabatic (i.e., potential temperature increases with height) at top, where exchange of air between the ABL and the free troposphere occurs
- wind speeds are sub-geostrophic in mixed layer, crossing isobars because of turbulent drag
Residual Layer
- disconnected from boundary layer and Earth’s surface
- neutrally stratified, i.e. it has small but near-equal turbulence in all directions
- contains moisture and trace atmospheric constituents from the day before
Stable Boundary Layer

- statically stable with weaker turbulence that occurs sporadically
- winds aloft may increase to supergeostrophic speeds (low-level jet or nocturnal jet)
- stability tends to suppress turbulence, except for occasional shear-generated turbulence caused by the low-level jet
PBL STRUCTURE OVER HIGH PRESSURE REGIONS

The **Surface layer** is a very shallow region close to the ground (up to 10% of PBL). It is characterized by a superadiabatic lapse rate, moisture decrease with height and strong wind shear.[2] Almost all wind shear and all the potential temperature gradient in the CBL are confined in the surface layer.

**Microlayer** (or **Interfacial layer**) is identifies as the few centimeters of air closest to the ground, where molecular transport dominated over turbulent transport.

The **Entrainment zone** (or **Capping inversion**) is about 40% of the depth of the PBL. It is the region of statically stable air at the top of the mixed layer, where there is entrainment of free atmosphere air downward and overshooting thermals upward. It inhibits mixing and confines pollutants.
1. **During daytime**: radiation coming from the sun passes through the atmosphere. The soil warms up and a mixed layer forms and grows during the morning.
2. **One or two hours before sunset**: the surface emits more energy than it receives, and convective turbulence is no longer fed by the ground.
3. **During nighttime**: the ground continues to radiate more infrared energy upward than it receives from the overlying atmosphere. The air above it gets colder and colder, generating a layer of cold (denser) air below a layer of warmer (lighter) air, giving rise to the stable stratified layer.
Turbulence in the daytime boundary layer is maintained primarily by sensible heating at the surface, which results in parcels of air that are warmer than their surroundings. The turbulent motion in the convective boundary layer is organized into long-lived updrafts and downdrafts that extend through the depth of the boundary layer and are carried by the mean wind.

The updrafts consist of accelerating parcels, while the downdrafts are caused by compensating downward motion. Thus, the velocities in updrafts are higher than those in downdrafts; mass balance requires that the horizontal area occupied by downdrafts is higher than that of updrafts.
CONVECTIVE PBL (2)

The potential temperature is super-adiabatic close to the surface: the potential temperature decreases with height. Above a tenth of the mixed layer height, the potential temperature is relatively uniform because of vigorous vertical mixing. The layer above the mixed layer can be stably stratified. The velocity profile in the daytime boundary layer is relatively flat in the mixed layer. The rapid change in velocity at the top of the boundary layer reflects the fact that the velocity is vertically mixed below the top.
Assumption: the sensible heat input into the atmosphere modifies the potential temperature in the mixed layer.

Consider a mixed layer that grows by eroding a layer with a stable potential temperature gradient.
AC → initial temperature profile
BC → potential temperature after sensible heating has occurred over a time, T, since sunrise
AB → temperature change at the surface
triangle ABC → modification of the energy of the PBL
\( \gamma \rightarrow \text{potential temperature gradient of AC} \)
\( \Delta \theta \rightarrow \text{temperature change in AB} \)

The energy equivalent of the triangle ABC can be written as:

\[
\text{Energy in ABC} = \rho c_p \frac{1}{2} \Delta \theta z_i
\]

Noticing that \( \Delta \theta = \gamma z_i \) we can equate this energy to the sensible heat flux integrated over \( T \) to obtain
Noticing that $\Delta \theta = \gamma z_i$ we can equate this energy to the sensible heat flux integrated over $T$ to obtain

$$\rho c_p \frac{1}{2} \gamma z_i^2 = \int_0^T H(t) dt$$

where $H(t)$ is the time-varying sensible heat flux. For simplicity, if we assume that the sensible heat flux increases linearly with time, we obtain the following expression for the mixed layer height:

$$z_i^2 = \frac{H_{max} T}{\gamma \rho c_p}$$

where $H_{max}$ is the maximum heat flux.
HEIGHT OF THE CONVECTIVE PBL (4)

\[ z_i^2 = \frac{H_{\text{max}}T}{\gamma \rho c_p} \]

Example:

\[ \frac{H_{\text{max}}}{\rho c_p} = 0.3 \, \frac{m}{sK} \]

\[ T = 6h \]

\[ \gamma = 5K/1000 \, m \]

We obtain \( z_i \approx 1000 \, m \)

Note that the boundary layer height increases with time as long as the heat flux is positive.
TURBULENT VELOCITIES IN THE CONVECTIVE PBL

Different formulations for the turbulent velocities have been proposed, basing on the *free convection velocity scale*

\[ u_f = \left( \frac{g}{T_s} Q_0 z \right)^{1/3} \]

\[ \sigma_w = 1.3 u_* \quad \text{for } z \leq 0.1 z_i \]

\[ \sigma_w = 1.3 (u_*^3 + u_f^3)^{1/3} \quad \text{for } z(1.3 u_*) \leq z \leq z(u_f) \]

Between 0.1\( z_i \) and close to the top of the mixed layer, \( \sigma_w \) associated with buoyancy production of turbulence is proportional to the *convective velocity scale* given by

\[ w_* = \left( \frac{g}{T_s} Q_0 z_i \right)^{1/3} \]

where \( z_i \) is the mixed layer height. Then, we find that

\[ \sigma_w = \sigma_v = \sigma_u \cong 0.6 w_* \]
When the sun sets, turbulence energy production by buoyancy comes to a stop. Over a period of an hour, the turbulence in the mixed layer collapses, and shear becomes the primary mechanism for the production of turbulence. Because the ground is initially warmer than the atmosphere, the thermal radiation leaving the ground exceeds that being supplied by the atmosphere. This deficit leads to a cooling of the ground.

Initially, both the sensible heat flux and the ground heat flux are directed away from the earth’s surface. The surface cools rapidly, and a point is reached at which the ground becomes colder than the layers above in the atmosphere. At this stage, the heat flux from the atmosphere is directed toward the earth’s surface.

This process is referred to as the formation of a radiation-induced surface inversion — the temperature (and the potential temperature) increases with height.
The mean wind, as obtained experimentally, can be written as

\[
U(z) = \frac{u^*}{k} \left[ \ln \left( \frac{z}{z_0} \right) - \psi_m \left( \frac{z}{L} \right) + \psi_m \left( \frac{z_0}{L} \right) \right]
\]

where

\[
\psi_m \left( \frac{z}{L} \right) = -17 \left[ 1 - \exp \left( -0.29 \frac{z}{L} \right) \right]
\]

No similar equations are available for the variation of temperature through the depth of the boundary layer.
Most expressions for the height of the stable boundary layer, which we denote by \( h \), are based on dimensional analysis backed by relatively weak physical arguments.

One of the most famous formulations, educed by Nieuwstadt’s (1981) is the following:

\[
h = u_* t \left[ \frac{L/h}{a + bL/h} \right]
\]

where \( a \) and \( b \) are empirical constants.

The problem with diagnostic equations is that the height of the boundary layer reacts instantaneously to \( u_* \) and \( L \). This means that \( h \) will drop suddenly (and unrealistically) if the wind speed, and thus \( u_* \) decreases quickly.
One way of getting around this problem is to allow the boundary layer to have some inertia. This is done by using the following equation to estimate the time evolution of $h$:

$$\frac{dh}{dt} = \frac{h_d - h}{\tau}$$

where $h_d$ is the estimate given by the diagnostic equation, and $\tau$ is the timescale, given by

$$\tau = \frac{\beta h}{u_*}$$

where $\beta$ is an empirical constant.

When $h_d = h$, $h$ does not change.

If $h_d$ increases suddenly in response to an increase in wind speed, $dh/dt$ becomes positive, so that $h$ will grow toward $h_d$; the time of reaction is proportional to $h/u_*$. This means that if either $h$ is large, or $u_*$ is small, $h$ reacts slowly to changes in $h_d$. 
TURBULENT VELOCITIES IN THE STABLE PBL

The stable potential temperature gradient suppresses the production of turbulence because it opposes vertical motion. Under these circumstances, shear production of turbulence is matched by the destruction associated with the stable temperature gradient and viscous dissipation. While we do know that the levels of turbulence in the stable boundary layer are low, we are not in a good position to characterize the variation of these levels as a function of height.

The parameterization that is sometimes used to estimate $\sigma_w$ is that of Nieuwstadt (1984):

$$\sigma_w^2 = 1.7 u_*^2 \left( 1 - \frac{z}{h} \right)$$

The horizontal turbulent velocities, $\sigma_u$ and $\sigma_v$, in the stable boundary layer do not appear to be related to micrometeorological variables. They are affected by mesoscale flows and local topography, which are difficult to characterize using models. In the absence of measurements, a value of $\sigma_v$ of 1 m/s can be used.
An example of a deep well mixing boundary layer in the Front range area of the Rockies, shown in Skew-T diagram.
An example morning sounding showing the surface inversion (stable) layer that developed due to night-time surface cooling. Such a shallow stable layer can usually be quickly removed after sunrise.
Phenomena, such as turbulence, with space scales smaller than about 2 km and with time scales shorter than 1 h are classified as **microscale**. The study of small-scale phenomena is named **micrometeorology**.
Micrometeorology is typically studies using:

- **stochastic** models, are a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. The random variation is usually based on fluctuations observed in historical data for a selected period using standard time-series techniques and deal with the average statically effects of the eddies.

- **similarity theory** involved the common-behavior exhibited by many empirically-observed phenomena of properly scaled.

- **phenomenological methods** permit to investigate large size structures (as thermals) in a deterministic manner.

Alternative approaches regard **numerical** and **laboratory simulations**.
The **urban boundary layer (UBL)** is the part of ABL in which most of the Earth’s population live and is one of the most complex and least understood topic in environmental fluid mechanics.

UBL characteristics are strictly linked to weather conditions, urban texture and exchanges of mass, momentum, heat and moisture between the urban canopy and the overlying air layers.
- *urban canopy layer* (UCL): the layer between the average height of the buildings and the surface
- *roughness sublayer* (RSL): the layer adjacent to the surface wherein the flow is strongly influenced by individual roughness elements and, hence, shows a full three-dimensional structure
- *constant flux layer* (CFL): extends from the top of the roughness sublayer to a few hundred meters above the surface. It is characterized by a nearly constant distribution of turbulence fluxes and the logarithmic velocity profile
**Wind profile over flat terrain**

\[ U(z) = \frac{u_*}{k} \ln \left( \frac{z}{z_0} \right) \]

- \( z \) height
- \( u_* \) friction velocity (i.e. the scaling velocity, which is related to the drag force at the surface)
- \( k=0.41 \) von Karman constant
- \( z_0 \) aerodynamic roughness length
### AERODYNAMIC ROUGHNESS LENGTH $z_0$

<table>
<thead>
<tr>
<th>Class</th>
<th>Terrain</th>
<th>$z_0$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>muddy terrains, wetlands, icepack</td>
<td>$10^{-5} \div 3 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>water areas*</td>
<td>$3 \times 10^{-5} \div 0.0002$</td>
</tr>
<tr>
<td>II</td>
<td>sand</td>
<td>$0.0002 \div 0.001$</td>
</tr>
<tr>
<td>III</td>
<td>airport runway areas, mown grass</td>
<td>$0.001 \div 0.01$</td>
</tr>
<tr>
<td>IV</td>
<td>farmland/airports with very few trees, buildings, etc.</td>
<td>$0.01 \div 0.04$</td>
</tr>
<tr>
<td>V</td>
<td>many trees and/or bushes</td>
<td>$0.04 \div 0.1$</td>
</tr>
<tr>
<td>VI</td>
<td>forecasts, suburbs</td>
<td>$0.1 \div 1$</td>
</tr>
<tr>
<td>VII</td>
<td>cities</td>
<td>$1 \div 4$</td>
</tr>
</tbody>
</table>

* air and sea form a dynamically coupled system, the determination of the roughness length of open sea and water surfaces is usually obtained by models taking into account shapes and dimension of the waves.

Wind profile over complex terrain

\[ U(z) = \frac{u_*}{k} \ln \left( \frac{z - d_0}{z_0} \right) \]

\( d_0 \) zero displacement height, i.e. the effective height of the ground due to the vertical flow displacement through the canopy
<table>
<thead>
<tr>
<th>Terrain</th>
<th>$d_0$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>not available</td>
</tr>
<tr>
<td>sand</td>
<td>not available</td>
</tr>
<tr>
<td>grass</td>
<td>0.07±0.66</td>
</tr>
<tr>
<td>crops</td>
<td>&lt; 3</td>
</tr>
<tr>
<td>orchards</td>
<td>&lt; 4</td>
</tr>
<tr>
<td>deciduous forests</td>
<td>&lt; 20</td>
</tr>
<tr>
<td>conifer forests</td>
<td>&lt; 30</td>
</tr>
</tbody>
</table>

Zero displacement height for different surfaces (from Monteith and Unsworth, 1990)
The shear in the approaching flow causes a downward flow over the lower portion of the upwind façade of the building and a “horse-shoe vortex” at the upwind base, which wraps around the building near the ground extending also further downstream.

The flow separates at the upwind edges, producing separation zones on the roof top and on the lateral sides of the building. At the downwind edges the flow separates again, producing a cavity region and the associated bow vortex. The flow there interacts with the current merging from the roof and the side walls as well as with the horseshoe vortex.
Sketch of the flow patterns around an isolated cubic building along a horizontal plane in the case of approaching wind a perpendicular to a façade and b forming an angle of 45° with the façade.
It is the basic geometric unit of urban areas and can be defined as a quasi-narrow street between buildings that line up continuously along both sides.

It is assumed as an archetype for more complex and realistic urban fabrics because of its simple and versatile representation.
FLOW REGIME IN 2D STREET CANYONS

ASPECT RATIO: 
AR = W/H

**SKIMMING FLOW**
AR < 1.5

**WAKE-INTERFERENCE REGIME**
1.5 ≤ AR ≤ 2.5

**ISOLATED ROUGHNESS FLOW**
AR > 2.5
Sketch of flow topology inside a 2D street canyon with external wind perpendicular to the street axis (from left).
Sketch of flow topology inside a 2D street canyon in the case of external wind not perpendicular to the street axis.

Streamlines of the velocity magnitude for different aspect ratios (AR) and relative height (R) of the buildings.
FLOW REGIME IN 3D STREET CANYONS (1)

**PLANAR AREA INDEX:** $\lambda_p = \frac{A_P}{A_T}$

Mean $\lambda_p$ value for European cities $\approx 0.33$

- **SKIMMING FLOW**
  $\lambda_p < 0.13$

- **WAKE-INTERFERENCE REGIME**
  $0.13 \leq \lambda_p \leq 0.35$

- **ISOLATED ROUGHNESS FLOW**
  $\lambda_p > 0.35$
a Sketch of archetypal an array of cubes usually adopted in CFD simulations and laboratory experiments
b instantaneous snapshot of the velocity field – obtained for a staggered building array with $\lambda_p = 0.25$
Streamlines along horizontal planes for aligned and staggered regular arrays of cubical buildings for two directions of the approaching flow.
Mean wind dominates horizontal transport.
Turbulence dominates vertical transport.
DISPERSION IN 2D STREET CANYON
DISPERSION AND VELOCITY IN 2D STREET CANYON
PBL – IMPORTANCE AND APPLICATIONS (1)

- It is the part of the atmosphere where we live
- Almost 50% of the turbulent kinetic energy is dissipated in the PBL
- Turbulent transport and advection move water and oxygen to/from plants
- Location of source and sink of many trace gases (e.g. CO₂, ozone, methane, water vapor), dust and pollutants
- It is very important for mesoscale weather conditions and climate change
- Thunderstorms and hurricane evolution are tied to the inflow of moist in PBL
- Crops are grown in the PBL, pollen is distributed by the PBL circulation
- Cloud nuclei are stirred into the air from the surface by PBL processes
PBL – IMPORTANCE AND APPLICATIONS (2)

- Air quality, atmospheric transport and diffusion of pollutants (local, urban and regional air quality)
- Mesoscale meteorology
- Urban planning, through flow dispersion around buildings, prediction and abatement of pollutant gases, prediction of road surface temperatures, icing
- Human comfort, heat waves and optimal design of structures and buildings
- Interaction with urban heat island
- Optimal design of wind turbines